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Über die Koeffizienten der schlichten Funktionen (IV)

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ÜBER DIE KOEFFIZIENTEN DER SCHLICHTEN FUNKTIONEN (IV).

KENITI KOSEKI

Diese Arbeit ist die Fortsetzung von meiner früheren Arbeit¹⁾ "ÜBER DIE KOEFFIZIENTEN DER SCHLICHTEN FUNKTIONEN (III)".

Ich will nun die Integralgleichung²⁾

$$\Im [6\kappa^3(s)e^{-s} + 12\kappa(s) \int_{-\infty}^0 2\kappa(s_1)e^{-s_1} \int_{-\infty}^{s_1} 2\kappa(s_2)e^{-s_2} ds_2 ds_1 + 6\kappa(s) \int_{-\infty}^s 2\kappa^2(s_1)e^{-2s_1} ds_1 + 4\kappa(s) \int_{-\infty}^0 2\kappa^2(s_1)e^{-2s_1} ds_1 + 12\kappa^2(s)e^{-s} \int_{-\infty}^0 2\kappa(s_1)e^{-s_1} ds_1 + 8\kappa^2(s)e^{-s} \int_{-\infty}^s 2\kappa(s_1)e^{-s_1} ds_1] = 0, \quad 0 \leq s < \infty, \quad (1)$$

unter keine Bedingung lösen.

Aus der Gleichung (1) folgt es ohne weiteres

$$\Im [6\kappa^3(s)e^{-2s} + \left\{ 12 \int_{-\infty}^0 2\kappa(s_1)e^{-s_1} \int_{-\infty}^{s_1} 2\kappa(s_2)e^{-s_2} ds_2 ds_1 + 4 \int_{-\infty}^0 2\kappa^2(s_1)e^{-2s_1} ds_1 \right\} \kappa(s) + 2\kappa(s) \int_{-\infty}^s 2\kappa^2(s_1)e^{-2s_1} ds_1 + 12\kappa^2(s)e^{-s} \int_{-\infty}^0 2\kappa(s_1)e^{-s_1} ds_1 - 4\kappa^2(s)e^{-s} \int_{-\infty}^s 2\kappa(s_1)e^{-s_1} ds_1] = 0.$$

Wir setzen

$$\begin{aligned} 12 \int_{-\infty}^0 2\kappa(s_1)e^{-s_1} \int_{-\infty}^{s_1} 2\kappa(s_2)e^{-s_2} ds_2 ds_1 + 4 \int_{-\infty}^0 2\kappa^2(s_1)e^{-2s_1} ds_1 &= A, \\ 12 \int_{-\infty}^0 2\kappa(s_1)e^{-s_1} ds_1 &= B, \\ e^{-s} = t, \quad \kappa(s) &= \kappa(t), \end{aligned}$$

so nimmt die obige Integralgleichung die folgende Form an

$$\Im [6\kappa^3(t)t^2 + A\kappa(t) + 2\kappa(t) \int_t^0 2\kappa^2(t_1)t_1 dt_1 + B\kappa^2(t)t - 4\kappa^2(t)t \int_t^0 2\kappa(t_1)dt_1] = 0, \quad 0 \leq t \leq 1. \quad (2)$$

Wir setzen $\kappa(t) = e^{i\varphi(t)}$, so ist es

$$(\kappa(t))' = \kappa(t)i\varphi'(t).$$

Indem wir die beiden Seiten der obigen Formel $(n-1)$ -malig differenzieren,

¹⁾ K. Koseki. ÜBER DIE KOEFFIZIENTEN DER SCHLICHTEN FUNKTIONEN (III). Math. Journ. Okay. Univ. Vol. II, NO. I, 1962.

²⁾ K. Koseki. a. a. O. S. 34.

bekommen wir

$$\frac{d^n \kappa(t)}{dt^n} = \sum_{j=0}^{n-1} \frac{d^j \kappa(t)}{dt^j} i_{n-1} C_j \frac{d^{n-j} \varphi(t)}{dt^{n-j}}, \quad n \geq 1. \quad (3)$$

Wir setzen $\frac{d^n \kappa(t)}{dt^n} = \kappa(t) a_n$ ($n=0, 1, 2, \dots$), so folgt aus (3)

$$\begin{aligned} a_n &= \sum_{j=0}^{n-1} a_j i_{n-1} C_j i \varphi^{(n-j)}(t) \\ &= a_0 i \varphi^{(n)}(t) + \sum_{p_1=1}^{n-1} a_{p_1} i_{n-1} C_{p_1} i \varphi^{(n-p_1)}(t) \\ &= a_0 i \varphi^{(n)}(t) + \sum_{p_1=1}^{n-1} \sum_{p_2=0}^{p_1-1} a_{p_2} i_{p_2-1} C_{p_2} i \varphi^{(p_2-p_1)}(t) i_{n-1} C_{p_1} i \varphi^{(n-p_1)}(t) \\ &= a_0 \left(i \varphi^{(n)}(t) + \sum_{p_1=1}^{n-1} i_{n-1} C_{p_1} i^2 \varphi^{(n-p_1)}(t) \varphi^{(p_1)}(t) \right) \\ &\quad + \sum_{p_2=1}^{n-2} \sum_{p_1=p_2+1}^{n-1} a_{p_2} i_{p_2-1} C_{p_2} i_{p_1-1} C_{p_1} i^2 \varphi^{(n-p_1)}(t) \varphi^{(p_1-p_2)}(t) \\ &= \dots \\ &= a_0 \left(i \varphi^{(n)}(t) + \sum_{p_1=1}^{n-1} i_{n-1} C_{p_1} i^2 \varphi^{(n-p_1)}(t) \varphi^{(p_1)}(t) \right. \\ &\quad + \sum_{p_2=1}^{n-2} \sum_{p_1=p_2+1}^{n-1} i_{n-1} C_{p_1} i_{p_1-1} C_{p_2} i^3 \varphi^{(n-p_1)}(t) \varphi^{(p_1-p_2)}(t) \varphi^{(p_2)}(t) + \dots \\ &\quad + \sum_{p_{n-2}=1}^2 \sum_{p_{n-3}=p_{n-2}+1}^3 \dots \sum_{p_1=p_2+1}^{n-1} i_{n-1} C_{p_1} i_{p_1-1} C_{p_2} \dots \\ &\quad \left. i_{p_{n-3}-1} C_{p_{n-2}} i^{n-1} \varphi^{(n-p_1)}(t) \varphi^{(p_1-p_2)}(t) \dots \varphi^{(p_{n-3}-p_{n-2})}(t) \varphi^{(p_{n-2})}(t) \right) \\ &\quad + \sum_{p_{n-1}=1}^1 \sum_{p_{n-2}=p_{n-1}+1}^2 \dots \sum_{p_1=p_2+1}^{n-1} a_1 i^{n-1} i_{n-1} C_{p_1} i_{p_1-1} C_{p_2} \dots i_{p_{n-2}-1} C_{p_1} \varphi^{(n-p_1)}(t) \dots \\ &\quad \varphi^{(p_{n-2}-p_{n-1})}(t) = a_0 \left\{ i \varphi^{(n)}(t) + \sum_{j=1}^{n-1} \left(\sum_{p_j=1}^{n-j} \sum_{p_{j-1}=p_j+1}^{n-j+1} \dots \sum_{p_1=p_2+1}^{n-1} i_{n-1} C_{p_1} i_{p_1-1} C_{p_2} \dots \right. \right. \\ &\quad \left. \left. i_{p_{j-1}-1} C_{p_j} i^{j+1} \varphi^{(n-p_1)}(t) \varphi^{(p_1-p_2)}(t) \dots \varphi^{(p_{j-1}-p_j)}(t) \varphi^{(p_j)}(t) \right) \right\}. \end{aligned}$$

Andererseits ist es offenbar $a_0=1$. So bekommen wir

$$\begin{aligned} a_n &= i \varphi^{(n)}(t) + \sum_{j=1}^{n-1} \left(\sum_{p_j=1}^{n-j} \sum_{p_{j-1}=p_j+1}^{n-j+1} \dots \sum_{p_1=p_2+1}^{n-1} i_{n-1} C_{p_1} i_{p_1-1} C_{p_2} \dots \right. \\ &\quad \left. i_{p_{j-1}-1} C_{p_j} i^{j+1} \varphi^{(n-p_1)}(t) \varphi^{(p_1-p_2)}(t) \dots \varphi^{(p_{j-1}-p_j)}(t) \varphi^{(p_j)}(t) \right) \quad (4) \end{aligned}$$

³⁾ Da $\lim_{t \rightarrow 0} \kappa(t)$ existiert, so kann $t=0$ eingeschlossen werden.

für $n \geq 2$.

Aus (4) folgt es ohne weiteres

$$\left. \begin{aligned} \frac{d^n \kappa^3(t)}{dt^n} &= \kappa^3(t) \left\{ i3\varphi^{(n)}(t) + \sum_{j=1}^{n-1} \left(\sum_{p_j=1}^{n-j} \sum_{p_{j-1}=p_j+1}^{n-j+1} \cdots \sum_{p_1=p_2+1}^{n-1} 3^{j+1} C_{p_1 p_{1-1}} C_{p_2} \cdots \right. \right. \\ &\quad \left. \left. p_{j-1-1} C_{p_j} i^{j+1} \varphi^{(n-p_1)}(t) \varphi^{(p_1-p_2)}(t) \cdots \varphi^{(p_{j-1}-p_j)}(t) \varphi^{(p_j)}(t) \right) \right\}, \quad n \geq 2, \\ \frac{d^{n-1} \kappa^3(t)}{dt^{n-1}} &= \kappa^3(t) \left\{ i3\varphi^{(n-1)}(t) + \sum_{j=1}^{n-2} \left(\sum_{p_j=1}^{n-j-1} \sum_{p_{j-1}=p_j+1}^{n-j} \cdots \sum_{p_1=p_2+1}^{n-2} 3^{j+1} C_{p_1 p_{1-1}} C_{p_2} \right. \right. \\ &\quad \left. \left. \cdots p_{j-1-1} C_{p_j} i^{j+1} \varphi^{(n-1-p_1)}(t) \varphi^{(p_1-p_2)}(t) \cdots \varphi^{(p_{j-1}-p_j)}(t) \varphi^{(p_j)}(t) \right) \right\}, \quad n \geq 3, \\ \frac{d^{n-2} \kappa^3(t)}{dt^{n-2}} &= \kappa^3(t) \left\{ i3\varphi^{(n-2)}(t) + \sum_{j=1}^{n-3} \left(\sum_{p_j=1}^{n-j-2} \sum_{p_{j-1}=p_j+1}^{n-j-1} \cdots \right. \right. \\ &\quad \left. \left. \sum_{p_1=p_2+1}^{n-3} 3^{j+1} C_{p_1 p_{1-1}} C_{p_2} \cdots p_{j-1-1} C_{p_j} i^{j+1} \varphi^{(n-2-p_1)}(t) \varphi^{(p_1-p_2)}(t) \cdots \right. \right. \\ &\quad \left. \left. \varphi^{(p_{j-1}-p_j)}(t) \varphi^{(p_j)}(t) \right) \right\}, \quad n \geq 4. \end{aligned} \right\} \quad (5)$$

Es ist nun

$$\frac{d \left(\kappa(t) \int_t^0 2\kappa^2(t_1) t_1 dt_1 \right)}{dt} = i\kappa(t) \varphi'(t) \int_t^0 2\kappa^2(t_1) t_1 dt_1 - 2\kappa^3(t) t.$$

Daher ist

$$\begin{aligned} \frac{d^n \left(\kappa(t) \int_t^0 2\kappa^2(t_1) t_1 dt_1 \right)}{dt^n} &= i \sum_{p_1=0}^{n-1} \varphi^{(n-p_1)}(t)_{n-1} C_{p_1} \frac{d^{p_1} \left(\kappa(t) \int_t^0 2\kappa^2(t_1) t_1 dt_1 \right)}{dt^{p_1}} \\ &\quad - 2 \sum_{p_1=0}^{n-1} (\kappa^3(t))^{(n-p_1-1)}_{n-1} C_{p_1} t^{(p_1)} = i \varphi^{(n)}(t) \kappa(t) \int_t^0 2\kappa^2(t_1) t_1 dt_1 + i \sum_{p_1=1}^{n-1} \varphi^{(n-p_1)}(t) \\ &\quad n_{-1} C_{p_1} \frac{d^{p_1} \left(\kappa(t) \int_t^0 2\kappa^2(t_1) t_1 dt_1 \right)}{dt^{p_1}} - 2(\kappa^3(t))^{n-1} t - 2(\kappa^3(t))^{n-2}_{n-1} C_1 = i \varphi^{(n)}(t) \kappa(t) \\ &\quad \int_t^0 2\kappa^2(t_1) t_1 dt_1 + i \sum_{p_1=1}^{n-1} \varphi^{(n-p_1)}(t)_{n-1} C_{p_1} \left[i \varphi^{(p_1)}(t) \kappa(t) \int_t^0 2\kappa^2(t_1) t_1 dt_1 + i \right. \\ &\quad \left. \sum_{p_2=1}^{p_1-1} \varphi^{(p_1-p_2)}(t)_{p_1-1} C_{p_2} \frac{d^{p_2} \left(\kappa(t) \int_t^0 2\kappa^2(t_1) t_1 dt_1 \right)}{dt^{p_2}} - 2(\kappa^3(t))^{(p_1-1)} t - 2(\kappa^3(t))^{(p_1-2)}_{p_1-1} C_1 \right] \\ &\quad - 2(\kappa^3(t))^{(n-1)} t - 2(\kappa^3(t))^{(n-2)}_{n-1} C_1 = i \varphi^{(n)}(t) \kappa(t) \int_t^0 2\kappa^2(t_1) t_1 dt_1 + i^2 \\ &\quad \sum_{p_1=1}^{n-1} n_{-1} C_{p_1} \varphi^{(n-p_1)}(t) \varphi^{(p_1)}(t) \kappa(t) \int_t^0 2\kappa^2(t_1) t_1 dt_1 - 2(\kappa^3(t))^{(n-1)} t - 2(\kappa^3(t))^{(n-2)}_{n-1} C_1 \\ &\quad - 2 \sum_{p_1=1}^{n-1} i_{n-1} C_{p_1} \varphi^{(n-p_1)}(t) \left\{ (\kappa^3(t))^{(p_1-1)} t + (\kappa^3(t))^{(p_1-2)}_{p_1-1} C_1 \right\} \end{aligned}$$

$$\begin{aligned}
& + \sum_{p_2=1}^{n-2} \sum_{p_1=p_2+1}^{n-1} i^2 {}_{n-1}C_{p_1 \ p_1-1} C_{p_2} \varphi^{(n-p_1)}(t) \varphi^{(p_1-p_2)}(t) \frac{d^{p_2} \left(\kappa(t) \int_t^0 2\kappa^2(t_1) t_1 dt_1 \right)}{dt^{p_2}} \\
& = i \varphi^{(n)}(t) \kappa(t) \int_t^0 2\kappa^2(t_1) t_1 dt_1 + i^2 \sum_{p_1=1}^{n-1} {}_{n-1}C_{p_1} \varphi^{(n-p_1)}(t) \varphi^{(p_1)}(t) \kappa(t) \\
& \int_t^0 2\kappa^2(t_1) t_1 dt_1 - 2(\kappa^3(t))^{(n-1)} t - 2(\kappa^3(t))^{(n-2)} {}_{n-1}C_1 - 2 \sum_{p_1=1}^{n-1} i {}_{n-1}C_{p_1} \varphi^{(n-p_1)}(t) \\
& \left\{ (\kappa^3(t))^{(p_1-1)} t + (\kappa^3(t))^{(p_1-2)} {}_{p_1-1}C_1 \right\} + \sum_{p_2=1}^{n-2} \sum_{p_1=p_2+1}^{n-1} i^2 {}_{n-1}C_{p_1 \ p_1-1} C_{p_2} \varphi^{(n-p_1)}(t) \varphi^{(p_1-p_2)}(t) \\
& \left[i \varphi^{(p_2)}(t) \kappa(t) \int_t^0 2\kappa^2(t_1) t_1 dt_1 + i \sum_{p_3=1}^{p_2-1} \varphi^{(p_2-p_3)}(t) {}_{p_2-1}C_{p_3} \frac{d^{p_3} \left(\kappa(t) \int_t^0 2\kappa^2(t_1) t_1 dt_1 \right)}{dt^{p_3}} \right. \\
& \left. - 2(\kappa^3(t))^{(p_2-1)} t - 2(\kappa^3(t))^{(p_2-2)} {}_{p_2-1}C_1 \right] = i \varphi^{(n)}(t) \kappa(t) \int_t^0 2\kappa^2(t_1) t_1 dt_1 + i^2 \\
& \sum_{p_1=1}^{n-1} {}_{n-1}C_{p_1} \varphi^{(n-p_1)}(t) \varphi^{(p_1)}(t) \kappa(t) \int_t^0 2\kappa^2(t_1) t_1 dt_1 + i^3 \\
& \sum_{p_2=1}^{n-2} \sum_{p_1=p_2+1}^{n-1} {}_{n-1}C_{p_1 \ p_1-1} C_{p_2} \varphi^{(n-p_1)}(t) \varphi^{(p_1-p_2)}(t) \varphi^{(p_2)}(t) \kappa(t) \int_t^0 2\kappa^2(t_1) t_1 dt_1 - 2(\kappa^3(t))^{(n-1)} t \\
& - 2(\kappa^3(t))^{(n-2)} {}_{n-1}C_1 - 2 \sum_{p_1=1}^{n-1} i {}_{n-1}C_{p_1} \varphi^{(n-p_1)}(t) \left\{ (\kappa^3(t))^{(p_1-1)} t + (\kappa^3(t))^{(p_1-2)} {}_{p_1-1}C_1 \right\} \\
& - 2 \sum_{p_2=1}^{n-2} \sum_{p_1=p_2+1}^{n-1} i^2 {}_{n-1}C_{p_1 \ p_1-1} C_{p_2} \varphi^{(n-p_1)}(t) \varphi^{(p_1-p_2)}(t) \left\{ (\kappa^3(t))^{(p_2-1)} t + (\kappa^3(t))^{(p_2-2)} {}_{p_2-1}C_1 \right\} \\
& + \sum_{p_3=1}^{n-3} \sum_{p_2=p_3+1}^{n-2} \sum_{p_1=p_2+1}^{n-1} i^3 {}_{n-1}C_{p_1 \ p_1-1} C_{p_2 \ p_2-1} C_{p_3} \varphi^{(n-p_1)}(t) \varphi^{(p_1-p_2)}(t) \varphi^{(p_2-p_3)}(t) \\
& \frac{d^{p_3} \left(\kappa(t) \int_t^0 2\kappa^2(t_1) t_1 dt_1 \right)}{dt^{p_3}} .
\end{aligned}$$

Indem wir dieses Verfahren wiederholen, bekommen wir

$$\begin{aligned}
& \frac{d^n \left(\kappa(t) \int_t^0 2\kappa^2(t_1) t_1 dt_1 \right)}{dt^n} = i \varphi^{(n)}(t) \kappa(t) \int_t^0 2\kappa^2(t_1) t_1 dt_1 + i^2 \\
& \sum_{p_1=1}^{n-1} {}_{n-1}C_{p_1} \varphi^{(n-p_1)}(t) \varphi^{(p_1)}(t) \kappa(t) \int_t^0 2\kappa^2(t_1) t_1 dt_1 + i^3 \\
& \sum_{p_2=1}^{n-2} \sum_{p_1=p_2+1}^{n-1} {}_{n-1}C_{p_1 \ p_1-1} C_{p_2} \varphi^{(n-p_1)}(t) \varphi^{(p_1-p_2)}(t) \varphi^{(p_2)}(t) \kappa(t) \int_t^0 2\kappa^2(t_1) t_1 dt_1 + \dots + \\
& i^n \sum_{p_{n-1}=1}^1 \sum_{p_{n-2}=p_{n-1}+1}^2 \dots \sum_{p_1=p_2+1}^{n-1} {}_{n-1}C_{p_1 \ p_1-1} C_{p_2} \dots C_{p_{n-2}-1} C_{p_{n-1}} \varphi^{(n-p_1)}(t) \varphi^{(p_1-p_2)}(t) \dots \\
& \varphi^{(p_{n-2}-p_{n-1})}(t) \varphi^{(p_{n-1})}(t) \kappa(t) \int_t^0 2\kappa^2(t_1) t_1 dt_1 - 2(\kappa^3(t))^{(n-1)} t - 2(\kappa^3(t))^{(n-2)} {}_{n-1}C_1 - 2
\end{aligned}$$

$$\sum_{p_1=1}^{n-1} i^{n-1} C_{p_1} \varphi^{(n-p_1)}(t) \left\{ (\kappa^3(t))^{(p_1-1)} t + (\kappa^3(t))^{(p_1-2)} p_{1-1} C_1 \right\} - \dots$$

$$- i^{n-1} \sum_{p_{n-1}=1}^1 \sum_{p_{n-2}=p_{n-1}+1}^2 \dots \sum_{p_1=p_2+1}^{n-1} C_{p_1} p_{1-1} C_{p_2} \dots p_{n-2-1} C_{p_{n-1}} \varphi^{(n-p_1)}(t) \varphi^{(p_1-p_2)}(t) \dots$$

$$\varphi^{(p_{n-2}-p_{n-1})}(t) \left\{ (\kappa^3(t))' t \right\}. \quad (6)$$

Aus (5) gewinnen wir

$$\frac{d^n \left(\kappa(t) \int_t^0 2\kappa^2(t_1) t_1 dt_1 \right)}{dt^n} \Big|_{t=0} = -2(\kappa^3(0))^{(n-3)} C_1 - 2 \sum_{j=1}^{n-2} \left(\sum_{p_j=2}^{n-j} \sum_{p_{j-1}=p_j+1}^{n-j+1} \dots \right.$$

$$\left. \sum_{p_1=p_2+1}^{n-1} i^{j-1} C_{p_1} p_{1-1} C_{p_2} \dots p_{j-1-1} C_{p_j} \varphi^{(n-p_1)}(0) \varphi^{(p_1-p_2)}(0) \dots \varphi^{(p_{j-1}-p_j)}(0) (\kappa^3(0))^{(p_j-2)} \right.$$

$$\left. \text{(für } n \geq 3). \right) \quad (7)$$

$$= -2 C_1 \kappa^3(0) \left\{ i 3 \varphi^{(n-2)}(0) + \sum_{j=1}^{n-3} \left(\sum_{p_j=1}^{n-j-2} \sum_{p_{j-1}=p_j+1}^{n-j-1} \dots \sum_{p_1=p_2+1}^{n-3} 3^{j+1} C_{p_1} p_{1-1} C_{p_2} \dots \right. \right.$$

$$\left. p_{j-1-1} C_{p_j} i^{j+1} \varphi^{(n-2-p_1)}(0) \varphi^{(p_1-p_2)}(0) \dots \varphi^{(p_{j-1}-p_j)}(0) \varphi^{(p_j)}(0) \right\} - 2 \kappa^3(0)$$

$$\sum_{j=1}^{n-4} \left(\sum_{p_j=4}^{n-j} \sum_{p_{j-1}=p_j+1}^{n-j+1} \dots \sum_{p_1=p_2+1}^{n-1} i^{j-1} C_{p_1} p_{1-1} C_{p_2} \dots p_{j-1-1} C_{p_j} \varphi^{(n-p_1)}(0) \varphi^{(p_1-p_2)}(0) \dots \right.$$

$$\left. \varphi^{(p_{j-1}-p_j)}(0) \left[i 3 \varphi^{(p_j-2)}(0) + \sum_{s=1}^{p_j-3} \left(\sum_{q_s=1}^{p_j-s-2} \sum_{q_{s-1}=q_s+1}^{p_j-s-1} \dots \sum_{q_1=q_2+1}^{p_j-3} 3^{s+1} C_{q_1} q_{1-1} C_{q_2} \dots \right. \right. \right.$$

$$\left. \left. q_{s-1-1} C_{q_s} i^{s+1} \varphi^{(p_j-2-q_1)}(0) \varphi^{(q_1-q_2)}(0) \dots \varphi^{(q_{s-1}-q_s)}(0) \varphi^{(q_s)}(0) \right] \right) - 2 \kappa^3(0) \sum_{j=1}^{n-2} \left(\sum_{p_j=2}^2 \sum_{p_{j-1}=3}^{n-j+1} \dots \right.$$

$$\left. \sum_{p_1=p_2+1}^{n-1} i^{j-1} C_{p_1} p_{1-1} C_{p_2} \dots p_{j-1-1} C_{p_j} \varphi^{(n-p_1)}(0) \varphi^{(p_1-p_2)}(0) \dots \varphi^{(p_{j-1}-p_j)}(0) \right) - 2 \kappa^3(0)$$

$$\sum_{j=1}^{n-3} \left(\sum_{p_j=3}^3 \sum_{p_{j-1}=4}^{n-j+1} \dots \sum_{p_1=p_2+1}^{n-1} i^{j-1} C_{p_1} p_{1-1} C_{p_2} \dots p_{j-1-1} C_{p_j} \varphi^{(n-p_1)}(0) \varphi^{(p_1-p_2)}(0) \dots \right.$$

$$\left. \varphi^{(p_{j-1}-p_j)}(0) \right) \quad \text{(für } n \geq 5). \quad (8)$$

Wir wollen nun $\frac{d^n \left(\kappa^2(t) \int_t^0 2\kappa(t_1) dt_1 \right)}{dt^n}$ ausrechnen.

$$\frac{d^n \left(\kappa^2(t) \int_t^0 2\kappa(t_1) dt_1 \right)}{dt^n} = \frac{d^n \kappa^2(t) \int_t^0 2\kappa(t_1) dt_1}{dt^n} t + C_1 \frac{d^{n-1} \kappa^2(t) \int_t^0 2\kappa(t_1) dt_1}{dt^{n-1}}. \quad (9)$$

Es ist aber

$$\frac{d \left(\kappa^2(t) \int_t^0 2\kappa(t_1) dt_1 \right)}{dt} = 2\kappa^2(t) i \varphi'(t) \int_t^0 2\kappa(t_1) dt_1 - 2\kappa^3(t).$$

Daraus folgt ohne weiteres

$$\begin{aligned}
 & \frac{d^n \left(\kappa^2(t) \int_t^0 2\kappa(t_1) dt_1 \right)}{dt^n} = 2i \sum_{p_1=0}^{n-1} \varphi^{(n-p_1)}(t) {}_{n-1}C_{p_1} \frac{d^{p_1} \left(\kappa^2(t) \int_t^0 2\kappa(t_1) dt_1 \right)}{dt^{p_1}} - 2(\kappa^3(t))^{(n-1)} \\
 & = 2i \varphi^{(n)}(t) \kappa^2(t) \int_t^0 2\kappa(t_1) dt_1 + 2i \sum_{p_1=1}^{n-1} \varphi^{(n-p_1)}(t) {}_{n-1}C_{p_1} \frac{d^{p_1} \left(\kappa^2(t) \int_t^0 2\kappa(t_1) dt_1 \right)}{dt^{p_1}} \\
 & \quad - 2(\kappa^3(t))^{(n-1)} = 2i \varphi^{(n)}(t) \kappa^2(t) \int_t^0 2\kappa(t_1) dt_1 + 2i \sum_{p_1=1}^{n-1} {}_{n-1}C_{p_1} \varphi^{(n-p_1)}(t) \\
 & \quad \left[2i \varphi^{(p_1)}(t) \kappa^3(t) \int_t^0 2\kappa(t_1) dt_1 + 2i \sum_{p_2=1}^{p_1-1} {}_{p_1-1}C_{p_2} \varphi^{(p_1-p_2)}(t) \frac{d^{p_2} \left(\kappa^2(t) \int_t^0 2\kappa(t_1) dt_1 \right)}{dt^{p_2}} \right. \\
 & \quad \left. - 2(\kappa^3(t))^{(p_1-1)} \right] - 2(\kappa^3(t))^{(n-1)} \\
 & = 2i \varphi^{(n)}(t) \kappa^2(t) \int_t^0 2\kappa(t_1) dt_1 + (2i)^2 \sum_{p_1=1}^{n-1} {}_{n-1}C_{p_1} \varphi^{(n-p_1)}(t) \varphi^{(p_1)}(t) \kappa^2(t) \\
 & \quad \int_t^0 2\kappa(t_1) dt_1 - 2(\kappa^3(t))^{(n-1)} - 2 \sum_{p_1=1}^{n-1} 2i {}_{n-1}C_{p_1} \varphi^{(n-p_1)}(t) (\kappa^3(t))^{(p_1-1)} + (2i)^2 \\
 & \quad \sum_{p_2=1}^{n-2} \sum_{p_1=p_2+1}^{n-1} {}_{n-1}C_{p_1} {}_{p_1-1}C_{p_2} \varphi^{(n-p_1)}(t) \varphi^{(p_1-p_2)}(t) \frac{d^{p_2} \left(\kappa^2(t) \int_t^0 2\kappa(t_1) dt_1 \right)}{dt^{p_2}} \\
 & = 2i \varphi^{(n)}(t) \kappa^2(t) \int_t^0 2\kappa(t_1) dt_1 + \sum_{p_1=1}^{n-1} (2i)^2 {}_{n-1}C_{p_1} \varphi^{(n-p_1)}(t) \varphi^{(p_1)}(t) \kappa^2(t) \int_t^0 2\kappa(t_1) dt_1 + \dots \\
 & \quad + \sum_{p_{n-1}=1}^1 \sum_{p_{n-2}=p_{n-1}+1}^2 \dots \sum_{p_1=p_2+1}^{n-1} (2i)^n {}_{n-1}C_{p_1} {}_{p_1-1}C_{p_2} \dots {}_{p_{n-2}-1}C_{p_{n-1}} \varphi^{(n-p_1)}(t) \varphi^{(p_1-p_2)}(t) \dots \\
 & \quad \varphi^{(p_{n-2}-p_{n-1})}(t) \varphi^{(p_{n-1})}(t) \kappa^2(t) \int_t^0 2\kappa(t_1) dt_1 - 2(\kappa^3(t))^{(n-1)} - 2 \\
 & \quad \sum_{p_1=1}^{n-1} 2i {}_{n-1}C_{p_1} \varphi^{(n-p_1)}(t) (\kappa^3(t))^{(p_1-1)} - \dots - 2 \sum_{p_{n-1}=1}^1 \sum_{p_{n-2}=p_{n-1}+1}^2 \dots \\
 & \quad \sum_{p_1=p_2+1}^{n-1} (2i)^{n-1} {}_{n-1}C_{p_1} {}_{p_1-1}C_{p_2} \dots {}_{p_{n-2}-1}C_{p_{n-1}} \varphi^{(n-p_1)}(t) \dots \varphi^{(p_{n-2}-p_{n-1})}(t) (\kappa^3(t))'. \quad (10)
 \end{aligned}$$

Daher ist es

$$\begin{aligned}
 & \frac{d^n \left(\kappa^2(t) \int_t^0 2\kappa(t_1) dt_1 \right)}{dt^n} \Big|_{t=0} = -2(\kappa^3(0))^{(n-1)} - 2 \sum_{j=1}^{n-1} \left(\sum_{p_j=1}^{n-j} \sum_{p_{j-1}=p_j+1}^{n-j+1} \dots \right. \\
 & \quad \sum_{p_1=p_2+1}^{n-1} (2i)^j {}_{n-1}C_{p_1} {}_{p_1-1}C_{p_2} \dots {}_{p_{j-1}-1}C_{p_j} \varphi^{(n-p_1)}(0) \varphi^{(p_1-p_2)}(0) \dots \\
 & \quad \left. \varphi^{(p_{j-1}-p_j)}(0) (\kappa^3(0))^{(p_j-1)} \right) \quad (\text{für } n \geq 2). \quad (11)
 \end{aligned}$$

Aus (9) und (11) gewinnen wir

$$\begin{aligned}
 & \frac{d^n \left(\kappa^2(t) \int_t^0 2\kappa(t_1) dt_1 \right)}{dt^n} \Big|_{t=0} = {}_n C_1 \left[-2(\kappa^3(0))^{(n-2)} - 2 \sum_{j=1}^{n-2} \left(\sum_{p_j=1}^{n-j-1} \sum_{p_{j-1}=p_j+1}^{n-j} \dots \right. \right. \\
 & \sum_{p_1=p_2+1}^{n-2} (2i)^j {}_{n-2} C_{p_1 \ p_{1-1}} C_{p_2} \dots \dots p_{j-1-1} C_{p_j} \varphi^{(n-1-p_1)}(0) \varphi^{(p_1-p_2)}(0) \dots \\
 & \left. \left. \varphi^{(p_{j-1}-p_j)}(0) (\kappa^3(0))^{(p_j-1)} \right) \right] \quad (\text{für } n \geq 3) \quad (12) \\
 & = -2 {}_n C_1 \kappa^3(0) \left\{ i3 \varphi^{(n-2)}(0) + \sum_{j=1}^{n-3} \left(\sum_{p_j=1}^{n-j-2} \sum_{p_{j-1}=p_j+1}^{n-j-1} \dots \sum_{p_1=p_2+1}^{n-3} 3^{j+1} {}_{n-3} C_{p_1 \ p_{1-1}} C_{p_2} \dots \right. \right. \\
 & p_{j-1-1} C_{p_j} i^{j+1} \varphi^{(n-2-p_1)}(0) \varphi^{(p_1-p_2)}(0) \dots \varphi^{(p_{j-1}-p_j)}(0) \varphi^{(p_j)}(0) \left. \right\} - 2 {}_n C_1 \kappa^3(0) \sum_{j=1}^{n-4} \left(\sum_{p_j=3}^{n-j-1} \right. \\
 & \sum_{p_{j-1}=p_j+1}^{n-j} \dots \sum_{p_1=p_2+1}^{n-2} (2i)^j {}_{n-2} C_{p_1 \ p_{1-1}} C_{p_2} \dots p_{j-1-1} C_{p_j} \varphi^{(n-1-p_1)}(0) \varphi^{(p_1-p_2)}(0) \dots \\
 & \varphi^{(p_{j-1}-p_j)}(0) \left[i3 \varphi^{(p_j-1)}(0) + \sum_{s=1}^{p_j-2} \left(\sum_{q_s=1}^{p_j-1-s} \sum_{q_{s-1}=q_s+1}^{p_j-s} \dots \sum_{q_1=q_2+1}^{p_j-2} 3^{s+1} {}_{p_j-2} C_{q_1 \ q_{1-1}} C_{q_2} \right. \right. \\
 & \dots q_{s-1-1} C_{q_s} i^{s+1} \varphi^{(p_j-1-q_1)}(0) \varphi^{(q_1-q_2)}(0) \dots \varphi^{(q_{s-1}-q_s)}(0) \varphi^{(q_s)}(0) \left. \right) \left. \right] - 2 {}_n C_1 \kappa^3(0) \\
 & \sum_{j=1}^{n-2} \left(\sum_{p_j=1}^1 \sum_{p_{j-1}=2}^{n-j} \dots \sum_{p_1=p_2+1}^{n-2} (2i)^j {}_{n-2} C_{p_1 \ p_{1-1}} C_{p_2} \dots p_{j-1-1} C_{p_j} \varphi^{(n-1-p_1)}(0) \varphi^{(p_1-p_2)}(0) \dots \right. \\
 & \left. \varphi^{(p_{j-1}-1)}(0) \right) - 2 {}_n C_1 \kappa^3(0) \sum_{j=1}^{n-3} \left(\sum_{p_j=3}^2 \sum_{p_{j-1}=3}^{n-j} \dots \sum_{p_1=p_2+1}^{n-2} (2i)^j {}_{n-2} C_{p_1 \ p_{1-1}} C_{p_2} \dots \right. \\
 & p_{j-1-1} C_{p_j} \varphi^{(n-1-p_1)}(0) \varphi^{(p_1-p_2)}(0) \dots \varphi^{(p_{j-1}-p_j)} 3i \varphi'(0) \left. \right) \quad (\text{für } n \geq 5). \quad (13)
 \end{aligned}$$

Wir setzen die (4), (5), (8) und (13) in (2) ein, so ergibt es sich

$$\begin{aligned}
 & \Im 12 {}_n C_2 \kappa^3(0) \left\{ i3 \varphi^{(n-2)}(0) + \sum_{j=1}^{n-3} \left(\sum_{p_j=1}^{n-j-2} \sum_{p_{j-1}=p_j+1}^{n-j-1} \dots \sum_{p_1=p_2+1}^{n-3} 3^{j+1} {}_{n-3} C_{p_1 \ p_{1-1}} C_{p_2} \dots \right. \right. \\
 & p_{j-1-1} C_{p_j} i^{j+1} \varphi^{(n-2-p_1)}(0) \varphi^{(p_1-p_2)}(0) \dots \varphi^{(p_{j-1}-p_j)}(0) \varphi^{(p_j)}(0) \left. \right\} + \Im \kappa(0) A \\
 & \left\{ i \varphi^{(n)}(0) + \sum_{j=1}^{n-1} \left(\sum_{p_j=1}^{n-j} \sum_{p_{j-1}=p_j+1}^{n-j+1} \dots \sum_{p_1=p_2+1}^{n-1} {}_{n-1} C_{p_1 \ p_{1-1}} C_{p_2} \dots \right. \right. \\
 & p_{j-1-1} C_{p_j} i^{j+1} \varphi^{(n-p_1)}(0) \varphi^{(p_1-p_2)}(0) \dots \varphi^{(p_{j-1}-p_j)}(0) \varphi^{(p_j)}(0) \left. \right\} - 4 {}_{n-1} C_1 \Im \kappa^3(0) \\
 & \left\{ i3 \varphi^{(n-2)}(0) + \sum_{j=1}^{n-3} \left(\sum_{p_j=1}^{n-j-2} \sum_{p_{j-1}=p_j+1}^{n-j-1} \dots \sum_{p_1=p_2+1}^{n-3} 3^{j+1} {}_{n-3} C_{p_1 \ p_{1-1}} C_{p_2} \dots \right. \right. \\
 & p_{j-1-1} C_{p_j} i^{j+1} \varphi^{(n-2-p_1)}(0) \varphi^{(p_1-p_2)}(0) \dots \varphi^{(p_{j-1}-p_j)}(0) \varphi^{(p_j)}(0) \left. \right\} - 4 \Im \kappa^3(0) \\
 & \sum_{j=1}^{n-4} \left(\sum_{p_j=4}^{n-j} \sum_{p_{j-1}=p_j+1}^{n-j+1} \dots \sum_{p_1=p_2+1}^{n-1} i^j {}_{n-1} C_{p_1 \ p_{1-1}} C_{p_2} \dots p_{j-1-1} C_{p_j} \varphi^{(n-p_1)}(0) \varphi^{(p_1-p_2)}(0) \dots \right.
 \end{aligned}$$

$$\begin{aligned}
& \varphi^{(p_{j-1}-p_j)}(0) \left[i3\varphi^{(p_j-2)}(0) + \sum_{s=1}^{p_j-3} \left(\sum_{q_s=1}^{p_j-s-2} \sum_{q_{s-1}=q_s+1}^{p_j-s-1} \cdots \sum_{q_1=q_2+1}^{p_j-3} 3^{s+1} \varphi^{(p_j-3)} C_{q_1} C_{q_1-1} C_{q_2} \cdots \right. \right. \\
& \left. \left. q_{s-1}-1 C_{q_s} i^{s+1} \varphi^{(p_j-2-q_1)}(0) \varphi^{(q_1-q_2)}(0) \cdots \varphi^{(q_{s-1}-q_s)}(0) \varphi^{(q_s)}(0) \right) \right] - 4\mathfrak{K}^3(0) \sum_{j=1}^{n-2} \left(\sum_{p_j=2}^2 \sum_{p_{j-1}=3}^{n-j+1} \right. \\
& \left. \cdots \sum_{p_1=p_2+1}^{n-1} i^j_{n-1} C_{p_1} C_{p_1-1} C_{p_2} \cdots C_{p_{j-1}-1} C_{p_j} \varphi^{(n-p_1)}(0) \varphi^{(p_1-p_2)}(0) \cdots \varphi^{(p_{j-1}-p_j)}(0) \right) \\
& - 4\mathfrak{K}^3(0) \sum_{j=1}^{n-3} \left(\sum_{p_j=3}^3 \sum_{p_{j-1}=4}^{n-j+1} \cdots \sum_{p_1=p_2+1}^{n-1} i^j_{n-1} C_{p_1} C_{p_1-1} C_{p_2} \cdots C_{p_{j-1}-1} C_{p_j} \varphi^{(n-p_1)}(0) \varphi^{(p_1-p_2)}(0) \cdots \right. \\
& \left. \varphi^{(p_{j-1}-p_j)}(0) 3i\varphi'(0) \right) + n C_1 \mathfrak{K} B_{\mathfrak{K}}^2(0) \left\{ i2\varphi^{(n-1)}(0) + \sum_{j=1}^{n-2} \left(\sum_{p_j=1}^{n-j-1} \sum_{p_{j-1}=p_j+1}^{n-j} \cdots \right. \right. \\
& \left. \left. \sum_{p_1=p_2+1}^{n-2} 2^{j+1}_{n-2} C_{p_1} C_{p_1-1} C_{p_2} \cdots C_{p_{j-1}-1} C_{p_j} i^{j+1} \varphi^{(n-1-p_1)}(0) \varphi^{(p_1-p_2)}(0) \cdots \right. \right. \\
& \left. \left. \varphi^{(p_{j-1}-p_j)}(0) \varphi^{(p_j)}(0) \right) \right\} + 8_n C_1 \mathfrak{K}^3(0) \left\{ i3\varphi^{(n-2)}(0) + \sum_{j=1}^{n-3} \left(\sum_{p_j=1}^{n-j-2} \sum_{p_{j-1}=p_j+1}^{n-j-1} \cdots \right. \right. \\
& \left. \left. \sum_{p_1=p_2+1}^{n-3} 3^{j+1}_{n-3} C_{p_1} C_{p_1-1} C_{p_2} \cdots C_{p_{j-1}-1} C_{p_j} i^{j+1} \varphi^{(n-2-p_1)}(0) \varphi^{(p_1-p_2)}(0) \cdots \right. \right. \\
& \left. \left. \varphi^{(p_{j-1}-p_j)}(0) \varphi^{(p_j)}(0) \right) \right\} + 8_n C_1 \mathfrak{K}^3(0) \sum_{j=1}^{n-4} \left(\sum_{p_j=3}^{n-j-1} \sum_{p_{j-1}=p_j+1}^{n-j} \cdots \sum_{p_1=p_2+1}^{n-2} (2i)^j_{n-2} C_{p_1} C_{p_1-1} C_{p_2} \cdots \right. \\
& \left. C_{p_{j-1}-1} C_{p_j} \varphi^{(n-1-p_1)}(0) \varphi^{(p_1-p_2)}(0) \cdots \varphi^{(p_{j-1}-p_j)}(0) \left[3i\varphi^{(p_j-1)}(0) + \sum_{s=1}^{p_j-2} \left(\sum_{q_s=1}^{p_j-1-s} \sum_{q_{s-1}=q_s+1}^{p_j-s} \cdots \right. \right. \right. \\
& \left. \left. \sum_{q_1=q_2+1}^{p_j-2} 3^{s+1}_{p_j-2} C_{q_1} C_{q_1-1} C_{q_2} \cdots C_{q_{s-1}-1} C_{q_s} i^{s+1} \varphi^{(p_j-1-q_1)}(0) \varphi^{(q_1-q_2)}(0) \cdots \varphi^{(q_{s-1}-q_s)}(0) \varphi^{(q_s)}(0) \right) \right] \\
& + 8_n C_1 \mathfrak{K}^3(0) \sum_{j=1}^{n-2} \left(\sum_{p_j=1}^1 \sum_{p_{j-1}=2}^{n-j} \cdots \sum_{p_1=p_2+1}^{n-2} (2i)^j_{n-2} C_{p_1} C_{p_1-1} C_{p_2} \cdots \right. \\
& \left. C_{p_{j-1}-1} C_{p_j} \varphi^{(n-1-p_1)}(0) \varphi^{(p_1-p_2)}(0) \cdots \varphi^{(p_{j-1}-p_j)}(0) \right) + 8_n C_1 \mathfrak{K}^3(0) \sum_{j=1}^{n-3} \left(\sum_{p_j=2}^2 \sum_{p_{j-1}=3}^{n-j} \cdots \right. \\
& \left. \sum_{p_1=p_2+1}^{n-2} (2i)^j_{n-2} C_{p_1} C_{p_1-1} C_{p_2} \cdots C_{p_{j-1}-1} C_{p_j} \varphi^{(n-1-p_1)}(0) \varphi^{(p_1-p_2)}(0) \cdots \right. \\
& \left. \varphi^{(p_{j-1}-p_j)}(0) 3i\varphi'(0) \right) = 0. \tag{14}
\end{aligned}$$

Diese Formel gilt für $n \geq 5$. Wenn $n = 4$ ist, muss es der Formel (14) an

$$\begin{aligned}
& \text{dem Glied } -4\mathfrak{K}^3(0) \sum_{j=1}^{n-4} \left(\sum_{p_j=4}^{n-j} \sum_{p_{j-1}=p_j+1}^{n-j+1} \cdots \sum_{p_1=p_2+1}^{n-1} i^j_{n-1} C_{p_1} C_{p_1-1} C_{p_2} \cdots \right. \\
& \left. C_{p_{j-1}-1} C_{p_j} \varphi^{(n-p_1)}(0) \varphi^{(p_1-p_2)}(0) \cdots \varphi^{(p_{j-1}-p_j)}(0) \left[i3\varphi^{(p_j-2)}(0) + \sum_{s=1}^{p_j-3} \left(\sum_{q_s=1}^{p_j-s-1} \cdots \right. \right. \right. \\
& \left. \left. \sum_{q_1=q_2+1}^{p_j-3} 3^{s+1}_{p_j-3} C_{q_1} C_{q_1-1} C_{q_2} \cdots C_{q_{s-1}-1} C_{q_s} i^{s+1} \varphi^{(p_j-2-q_1)}(0) \varphi^{(q_1-q_2)}(0) \cdots \right. \right. \\
& \left. \left. \varphi^{(q_{s-1}-q_s)}(0) \varphi^{(q_s)}(0) \right) \right] \right)
\end{aligned}$$

und dem $8, C_1 \mathfrak{K}^3(0) \sum_{j=1}^{n-4} \left(\sum_{p_j=3}^{n-j-1} \sum_{p_{j-1}=p_j+1}^{n-j} \dots \sum_{p_1=p_2+1}^{n-2} (2i)^{j-n-2} C_{p_1} p_{1-1} C_{p_2} \dots \right.$
 $p_{j-1-1} C_{p_j} \varphi^{(n-1-p_1)}(0) \varphi^{(p_1-p_2)}(0) \dots \varphi^{(p_{j-1}-p_j)}(0) \left[3i \varphi^{(p_j-1)}(0) + \sum_{s=1}^{p_j-2} \left(\sum_{q_s=1}^{p_j-1-s} \sum_{q_{s-1}=q_s+1}^{p_1-s} \dots \right. \right.$
 $\sum_{q_1=q_2+1}^{p_j-2} 3^{s+1} p_{j-2} C_{q_1} q_{1-1} C_{q_2} \dots q_{s-1-1} C_{q_s} i^{s+1} \varphi^{(p_j-1-q_1)}(0) \varphi^{(q_1-q_2)}(0) \dots$
 $\left. \left. \varphi^{(q_{s-1}-q_s)}(0) \varphi^{(q_s)}(0) \right) \right] \Bigg) \text{ fehlen. Wenn } n=3 \text{ ist, besteht es die folgende}$
Formel

$$\begin{aligned} & \mathfrak{S}12_n C_2 \mathfrak{r}^3(0) i 3 \varphi^{(n-2)}(0) + \mathfrak{S} \kappa(0) A \left\{ i \varphi^{(n)}(0) + \sum_{j=1}^{n-1} \left(\sum_{p_j=1}^{n-j} \sum_{p_{j-1}=1}^{n-j+1} \dots \right. \right. \\ & \left. \left. \sum_{p_1=p_2+1}^{n-1} n-1 C_{p_1} p_{1-1} C_{p_2} \dots p_{j-1-1} C_{p_j} i^{j+1} \varphi^{(n-p_1)}(0) \varphi^{(p_1-p_2)}(0) \dots \varphi^{(p_{j-1}-p_j)}(0) \varphi^{(p_j)}(0) \right) \right\} \\ & - 4_{n-1} C_1 \mathfrak{S} \kappa^3(0) i 3 \varphi^{(n-2)}(0) - 4 \mathfrak{S} \kappa^3(0) i_{n-2} C_2 \varphi^{(n-3)}(0) + {}_n C_1 \mathfrak{S} B \kappa^2(0) \left(i 2 \varphi^{(n-1)}(0) + \right. \\ & \left. 2^2 i^2 \varphi'(0) \right) + 8_n C_1 \mathfrak{S} \kappa^3(0) i 3 \varphi^{(n-2)}(0) + 8_n C_1 \mathfrak{S} \kappa^3(0) 2 i \varphi'(0) = 0. \end{aligned} \quad (15)$$

Wenn $n=2$ ist, so besteht es die Formel

$$\Im 12\kappa^3(0) + \Im \kappa(0) A \left(i\varphi''(0) + i^2\varphi'(0) \right) - 4_{n-1} C_1 \Im \kappa^3(0) + {}_n C_1 \Im B \kappa^2(0) i2\varphi'(0) + 8_n C_1 \Im \kappa^3(0) = 0. \quad (16)$$

Wir setzen nun $\varphi^{(k)}(0) = a_{k,0} + a_{k,1}\varphi'(0) + \dots + a_{k,k}\varphi^{(k)}(0) = \sum_{j=0}^k a_{k,j}\varphi^{(j)}(0)$,⁴⁾ so ergibt es sich aus (14), (15) und (16)

$$a_{n,n} = -\Im \kappa(0) A \sum_{j=1}^{n-1} \left(\sum_{p_j=1}^{n-j} \sum_{p_{j-1}=p_j+1}^{n-j+1} \cdots \sum_{p_1=p_2+1}^{n-1} {}^{n-1}C_{p_1} {}^{p_1-1}C_{p_2} \cdots \right. \\ \left. {}^{p_{j-1}-1}C_{p_j} i^{j+1} a_{n-p_1, n-p_1} a_{p_1-p_2, p_1-p_2} \cdots a_{p_{j-1}-p_j, p_{j-1}-p_j} a_{p_j, p_j} \Re(\kappa(0) A) \right). \quad (17)$$

Diese Formel gilt für $n \geq 2$. Wenn j gerade ist, ist es $\mathfrak{J}(\kappa(0)A i^{j+1}) = - (1)^{\frac{j}{2}} \mathfrak{R}(\kappa(0)A)$. Wenn j ungerade ist, ist es $\mathfrak{J}(\kappa(0)A i^{j+1}) = (-1)^{\frac{j+1}{2}} \mathfrak{S}(\kappa(0)A) = 0$. Der Einfachheit halber bezeichnen wir von jetzt an die $\kappa(0)$ nur mit κ . Dann ergibt es sich aus (17)

$$a_{n,n} = - \sum_{j=1}^{n-1} (-1)^{\frac{j}{2}} \left(\sum_{p_j=1}^{n-j} \sum_{p_{j-1}=p_j+1}^{n-j+1} \cdots \sum_{p_1=p_2+1}^{n-1} {}^{n-1}C_{p_1} {}^{p_1-1}C_{p_2} \cdots \right. \\ \left. {}^{p_{j-1}-1}C_{p_j} a_{n-p_1, n-p_1} a_{p_1-p_2, p_1-p_2} \cdots a_{p_{j-1}-p_j, p_{j-1}-p_j} a_{p_j, p_j} \right) = - \sum_{j=1}^{n-1} (-1)^{\frac{j}{2}} \\ \left(\sum_{p_1=j}^{n-1} \sum_{p_2=j-1}^{p_1-1} \cdots \sum_{p_j=1}^{p_{j-1}-1} {}^{n-1}C_{p_1} {}^{p_1-1}C_{p_2} \cdots {}^{p_{j-1}-1}C_{p_j} a_{n-p_1, n-p_1} a_{p_1-p_2, p_1-p_2} \cdots \right. \\ \left. a_{p_{j-1}-p_j, p_{j-1}-p_j} a_{p_j, p_j} \right), \quad (18)$$

⁴⁾ Diese Darstellung ist nicht notwendig eindeutig.

wo die Summe $\sum_{j=1}^{n-1}$ über alle positiven geraden ganzen Zahlen bis 1 zu $n-1$ zu erstrecken ist.

Wir setzen nun in der Formel (18)

$$\left. \begin{array}{l} n - p_1 = r_1 \\ p_1 - p_2 = r_2 \\ \vdots \\ p_{j-1} - p_j = r_j \\ p_j = r_{j+1} \end{array} \right\}$$

so gewinnen wir aus (18)

$$a_{n,n} = - \sum_{j=1}^{n-1} (-1)^{\frac{j}{2}} \sum_{r_1=1}^j \sum_{r_2=1}^j \cdots \sum_{r_{j+1}=1}^j \frac{(n-1)!}{(r_1-1)!(r_2-1)! \cdots (r_{j+1}-1)!} a_{r_1, r_1} a_{r_2, r_2} \cdots a_{r_{j+1}, r_{j+1}} \prod_{k=1}^j \frac{1}{n - r_1 - \cdots - r_k}, \quad (19)$$

wo die Summe $\sum_{r_1=1}^j \sum_{r_2=1}^j \cdots \sum_{r_{j+1}=1}^j$ über alle Systeme von positiven ganzen Zahlen r_1, r_2, \dots, r_{j+1} zu erstrecken ist, derart dass $r_1 + r_2 + \cdots + r_{j+1} = n$ ist.

Anderseits folgt es aus (2)

$$\Re \left[18x^3(t)t^2 + A\kappa(t) + 2x(t) \int_t^0 2\kappa^2(t_1)t_1 dt_1 + 2\kappa^2(t)tB - 8x^2(t)t \int_t^0 2x(t_1)dt_1 \right] \varphi'(t) + \Im \left[16x^3(t)t + B\kappa^2(t) - 4x^2(t) \int_t^0 2\kappa(t_1)dt_1 \right] = 0.$$

Daher ist für $t=0$

$$\varphi'(0) = - \frac{\Im B\kappa^2(0)}{\Re A\kappa(0)} = - \frac{\Im B\kappa^2}{\Re A\kappa}. \quad (20)$$

Da $\varphi'(0) = \varphi''(0)$ ist, können wir setzen $a_{1,1} = 1$ und $a_{1,0} = 0$.

Erstens behandeln wir den Fall, wo das Suffix n von $a_{n,n}$ ungerade ist. Wenn $j = n-1$ ist, so müssen alle r_1, r_2, \dots, r_{j+1} gleich 1 sein, da $r_1 + r_2 + \cdots + r_{j+1} = n$ ist. Daher ist

$$\frac{(n-1)!}{(r_1-1)!(r_2-1)! \cdots (r_{j+1}-1)!} a_{r_1, r_1} a_{r_2, r_2} \cdots a_{r_{j+1}, r_{j+1}} \prod_{k=1}^j \frac{1}{n - r_1 - \cdots - r_k} = (n-1)! \prod_{k=1}^{n-1} \frac{1}{n-k},$$

wenn $j = n-1$ ist. Daraus und aus (19) folgt es ohne weiteres

$$a_{n,n} = - (-1)^{\frac{n-1}{2}} (n-1)! \prod_{k=1}^{n-1} \frac{1}{n-k} - \sum_{j=1}^{n-2} (-1)^{\frac{j}{2}} \sum_{r_1=1}^j \sum_{r_2=1}^j \cdots \sum_{r_{j+1}=1}^j \frac{(n-1)!}{(r_1-1)! \cdots (r_{j+1}-1)!} a_{r_1, r_1} \cdots a_{r_{j+1}, r_{j+1}} \prod_{k=1}^j \frac{1}{n - r_1 - \cdots - r_k}. \quad (21)$$

Wenn $j \leq n-2$ ist, so muss mindestens eine unter r_1, r_2, \dots, r_{j+1} grösser als 1 sein. Wir nehmen nun an, dass $r_{j+1} > 1$ ist. Es ist dann

$$\begin{aligned}
 & (-1)^{\frac{j}{2}} \sum_{r_1=1}^j \sum_{r_2=1}^j \dots \sum_{r_j=1}^j \sum_{\substack{r_{j+1}>1 \\ (r_1+r_2+\dots+r_{j+1}=n, r_{j+1}>1)}} \frac{(n-1)!}{(r_1-1)! (r_2-1)! \dots (r_{j+1}-1)!} a_{r_1, r_1} \dots \\
 & a_{r_{j+1}, r_{j+1}} \prod_{k=1}^j \frac{1}{n-r_1-\dots-r_k} = (-1)^{\frac{j}{2}} \sum_{r_1=1}^j \sum_{r_2=1}^j \dots \\
 & \sum_{\substack{(r_1+\dots+r_j=n-r_{j+1})}} \frac{(n-1)!}{(r_1-1)! (r_2-1)! \dots (r_j-1)!} a_{r_1, r_1} a_{r_2, r_2} \dots a_{r_j, r_j} \times \\
 & - \sum_{j_2=1}^{r_{j+1}-1} (-1)^{\frac{j_2}{2}} \sum_{s_1=1}^{j_2} \sum_{s_2=1}^{j_2} \dots \sum_{\substack{s_{j_2+1}=1 \\ (s_1+s_2+\dots+s_{j_2+1}=r_{j+1})}} \frac{1}{(s_1-1)! \dots (s_{j_2+1}-1)!} a_{s_1, s_1} a_{s_2, s_2} \dots \\
 & a_{s_{j_2+1}, s_{j_2+1}} \prod_{k_2=1}^{j_2} \frac{1}{r_{j+1}-s_1-\dots-s_{k_2}} \prod_{k_1=1}^j \frac{1}{n-r_1-\dots-r_{k_1}}. \\
 \text{Daher ist es} \\
 & - \sum_{j=1}^{n-2} (-1)^{\frac{j}{2}} \sum_{r_1=1}^j \sum_{r_2=1}^j \dots \sum_{r_j=1}^j \sum_{\substack{r_{j+1}>1 \\ (r_1+r_2+\dots+r_{j+1}=n, r_{j+1}>1)}} \frac{(n-1)!}{(r_1-1)! (r_2-1)! \dots (r_{j+1}-1)!} a_{r_1, r_1} \dots \\
 & a_{r_{j+1}, r_{j+1}} \prod_{k=1}^j \frac{1}{n-r_1-\dots-r_k} = (-1)^{\frac{j}{2}} \sum_{j_2=2}^{n-1} (-1)^{\frac{j_2}{2}} \sum_{r_1=1}^{j_2} \sum_{r_2=1}^{j_2} \dots \sum_{\substack{r_{j_2+1}=1 \\ (r_1+\dots+r_{j_2+1}=n)}} \\
 & \frac{(n-1)!}{(r_1-1)! (r_2-1)! \dots (r_{j_2+1}-1)!} a_{r_1, r_1} a_{r_2, r_2} \dots a_{r_{j_2+1}, r_{j_2+1}} \prod_{j_1=1}^{j_2-1} \prod_{k_1=1}^{j_1} \frac{1}{n-r_1-\dots-r_{k_1}} \\
 & \prod_{k_2=1}^{j_2} \frac{1}{r_{j_2+1}+\dots+r_{j_2+1}}. \quad (22)
 \end{aligned}$$

Wir nehmen zweitens an, dass in der Formel (21) $r_j > 1$ und $r_{j+1} = 1$ ist. Es ist dann

$$\begin{aligned}
 & (-1)^{\frac{j}{2}} \sum_{r_1=1}^j \sum_{r_2=1}^j \dots \sum_{r_j=1}^j \sum_{\substack{r_{j+1}=1 \\ (r_1+r_2+\dots+r_{j+1}=n, r_{j+1}=1)}} \frac{(n-1)!}{(r_1-1)! \dots (r_{j+1}-1)!} a_{r_1, r_1} a_{r_2, r_2} \dots \\
 & a_{r_{j+1}, r_{j+1}} \prod_{k=1}^j \frac{1}{n-r_1-\dots-r_k} = (-1)^{\frac{j}{2}} \sum_{r_1=1}^j \sum_{r_2=1}^j \dots \sum_{\substack{r_{j-1}=1 \\ (r_1+\dots+r_{j-1}=n-1-r_j)}} \frac{(n-1)!}{(r_1-1)! \dots (r_{j-1}-1)!} a_{r_1, r_1} \\
 & \dots a_{r_{j-1}, r_{j-1}} \times (-1)^{\frac{j}{2}} \sum_{j_2=1}^{r_j-1} (-1)^{\frac{j_2}{2}} \sum_{s_1=1}^{j_2} \sum_{s_2=1}^{j_2} \dots \sum_{\substack{s_{j_2+1}=1 \\ (s_1+s_2+\dots+s_{j_2+1}=r_j)}} \frac{1}{(s_1-1)! \dots (s_{j_2+1}-1)!} a_{s_1, s_1} a_{s_2, s_2} \dots
 \end{aligned}$$

⁵⁾ Das Suffix j_2 beginnt in der Tat mit 4. Wenn $j_2=2$ ist, so ist $\sum_{j_1=1}^{j_2-1} = 0$.

$$a_{s_{j_2+1}, s_{j_2+1}} \prod_{k_1=1}^j \frac{1}{n-r_1 \cdots -r_{k_1}} \prod_{k_2=1}^{j_2} \frac{1}{r_j-s_1 \cdots -s_{k_2}}.$$

Daher ist es

$$\begin{aligned} & - \sum_{j=1}^{n-2} (-1)^{\frac{j}{2}} \sum_{r_1=1}^j \sum_{r_2=1}^j \cdots \sum_{r_j > 1, r_{j+1}=1} \frac{(n-1)!}{(r_1-1)! \cdots (r_{j+1}-1)!} a_{r_1, r_1} a_{r_2, r_2} \cdots \\ & \quad (r_1+r_2+\cdots+r_{j+1}=n, r_j > 1, r_{j+1}=1) \\ & a_{r_{j+1}, r_{j+1}} \prod_{k=1}^j \frac{1}{n-r_1 \cdots -r_k} = (-1)^2 \sum_{j_2=2}^{n-1} (-1)^{\frac{j_2}{2}} \sum_{r_1=1}^{j_2} \sum_{r_2=1}^{j_2} \cdots \sum_{r_{j_2-1}=1}^{j_2} \\ & \quad (r_1+r_2+\cdots+r_{j_2}=n-1) \\ & \frac{(n-1)!}{(r_1-1)! \cdots (r_{j_2}-1)!} a_{r_1, r_1} a_{r_2, r_2} \cdots a_{r_{j_2}, r_{j_2}} \prod_{j_1=1}^{j_2-1} \prod_{k_1=1}^{j_1-1} \frac{1}{1+r_{k_1+1}+\cdots+r_{j_2}} \\ & \prod_{k_2=j_1}^{j_2-1} \frac{1}{r_{k_2+1}+\cdots+r_{j_2}}. \end{aligned} \quad (23)$$

Wir nehmen drittens an, dass in der Formel (21) $r_{j-1} > 1$ und $r_j = r_{j+1} = 1$ ist. Es ist dann

$$\begin{aligned} & (-1)^{\frac{j}{2}} \sum_{r_1=1}^j \sum_{r_2=1}^j \cdots \sum_{r_{j-1} > 1, r_j=1, r_{j+1}=1} \frac{(n-1)!}{(r_1-1)! \cdots (r_{j+1}-1)!} a_{r_1, r_1} a_{r_2, r_2} \cdots \\ & \quad (r_1+r_2+\cdots+r_{j+1}=n, r_{j-1} > 1, r_j=r_{j+1}=1) \\ & a_{r_{j+1}, r_{j+1}} \prod_{k=1}^j \frac{1}{n-r_1 \cdots -r_k} = (-1)^{\frac{j}{2}} \sum_{r_1=1}^j \sum_{r_2=1}^j \cdots \sum_{r_{j-2}=1}^j \frac{(n-1)!}{(r_1-1)! \cdots (r_{j-2}-1)!} a_{r_1, r_1} \cdots \\ & \quad (r_1+r_2+\cdots+r_{j-2}=n-2-r_{j-1}) \\ & a_{r_{j-2}, r_{j-2}} \times (-1)^{\frac{r_{j-1}-1}{2}} \sum_{j_2=1}^{r_{j-1}-1} (-1)^{\frac{j_2}{2}} \sum_{s_1=1}^{j_2} \cdots \sum_{s_{j_2+1}=1}^{j_2} \frac{1}{(s_1-1)! \cdots (s_{j_2+1}-1)!} a_{s_1, s_1} \cdots \\ & \quad (s_1+\cdots+s_{j_2+1}=r_{j-1}) \\ & a_{s_{j_2+1}, s_{j_2+1}} \prod_{k_1=1}^j \frac{1}{n-r_1 \cdots -r_{k_1}} \prod_{k_2=1}^{j_2} \frac{1}{r_{j-1}-s_1 \cdots -s_{k_2}}. \end{aligned}$$

Daher ist es

$$\begin{aligned} & - \sum_{j=2}^{n-2} (-1)^{\frac{j}{2}} \sum_{r_1=1}^j \cdots \sum_{r_{j-1} > 1, r_j=1, r_{j+1}=1} \frac{(n-1)!}{(r_1-1)! \cdots (r_{j+1}-1)!} a_{r_1, r_1} \cdots \\ & \quad (r_1+r_2+\cdots+r_{j+1}=n, r_{j-1} > 1, r_j=r_{j+1}=1) \\ & a_{r_{j+1}, r_{j+1}} \prod_{k_1=1}^j \frac{1}{n-r_1 \cdots -r_{k_1}} = (-1)^2 \sum_{j_2=4}^{n-1} (-1)^{\frac{j_2}{2}} \sum_{r_1=1}^{j_2} \sum_{r_2=1}^{j_2} \cdots \sum_{r_{j_2-1}=1}^{j_2} \\ & \quad (r_1+r_2+\cdots+r_{j_2-1}=n-2) \\ & \frac{(n-1)!}{(r_1-1)! \cdots (r_{j_2-1}-1)!} a_{r_1, r_1} \cdots a_{r_{j_2-1}, r_{j_2-1}} \sum_{j_1=2}^{j_2-1} \alpha_2^{1,1} \prod_{k_2=j_1-1}^{j_2-2} \frac{1}{r_{k_2+1}+\cdots+r_{j_2-1}}, \end{aligned} \quad (24)$$

wo

$$\alpha_2^{1,1} = \begin{cases} \frac{1}{2!} & (\text{wenn } j_1=2 \text{ ist}) \\ \frac{1}{2!} \prod_{k_1=1}^{j_1-2} \frac{1}{2+r_{k_1+1}+\cdots+r_{j_2-1}} & (\text{wenn } j_1>2 \text{ ist}) \end{cases} \text{ ist.}$$

Dieses Verfahren wiederholen wir, so erhalten wir endlich

$$\begin{aligned} & (-1)^{\frac{j}{2}} \sum_{r_1 > 1} \sum_{r_2=1} \dots \sum_{\substack{r_{j+1}=1 \\ (r_1+r_2+\dots+r_{j+1}=n, r_2=\dots=r_{j+1}=1, r_1>1)}} \frac{(n-1)!}{(r_1-1)! \dots (r_{j+1}-1)!} a_{r_1, r_1} \dots a_{r_{j+1}, r_{j+1}} \prod_{k=1}^j \frac{1}{n-r_1-\dots-r_{k_2}} \\ & \frac{1}{n-r_1-\dots-r_{k_2}} = (-1)^{\frac{j}{2}} \sum_{r_1=n-j}^{n-1} \frac{(n-1)!}{(r_1-1)!} \times (-1)^{\sum_{j_2=1}^{r_1-1} j_2} (-1)^{\frac{j_2}{2}} \sum_{s_1=1}^{j_2} \dots \\ & \quad (s_1+\dots+s_{j_2+1}=r_1) \\ & \sum_{s_{j_2+1}=1}^{(r_1-1)!} \frac{(r_1-1)!}{(s_1-1)! \dots (s_{j_2+1}-1)!} a_{s_1, s_1} \dots a_{s_{j_2+1}, s_{j_2+1}} \prod_{k_1=1}^j \frac{1}{n-r_1-\dots-r_{k_1 k_2}} \prod_{k=1}^{j_2} \frac{1}{r_1-s_1-\dots-s_{k_2}}. \end{aligned}$$

Daher ist es

$$\begin{aligned} & - \sum_{j=n-3}^{n-2} (-1)^{\frac{j}{2}} \sum_{r_1 > 1} \sum_{r_2=1} \dots \sum_{\substack{r_{j+1}=1 \\ (r_1+\dots+r_{j+1}=n, r_1>1, r_2=\dots=r_{j+1}=1)}} \frac{(n-1)!}{(r_1-1)! \dots (r_{j+1}-1)!} a_{r_1, r_1} \dots \\ & a_{r_{j+1}, r_{j+1}} \prod_{k=1}^j \frac{1}{n-r_1-\dots-r_k} = (-1)^2 \sum_{j_2=n-2}^{n-1} (-1)^{\frac{j_2}{2}} \sum_{r_1=1}^{j_2} \sum_{r_2=1}^{j_2} \sum_{\substack{r_3=1 \\ (r_1+r_2+r_3=3)}} \frac{(n-1)!}{(r_1-1)! (r_2-1)! (r_3-1)!} a_{r_1, r_1} a_{r_2, r_2} a_{r_3, r_3} \frac{1}{(n-3)!} \frac{1}{(r_2+r_3)r_3}. \end{aligned} \quad (25)$$

Aus (21), (22), (23), (24) und (25) gewinnen wir sofort

$$\begin{aligned} a_{n,n} &= -(-1)^{\frac{n-1}{2}} \frac{(n-1)!}{(n-1)!} + (-1)^2 \sum_{j_2=2}^{n-1} (-1)^{\frac{j_2}{2}} \sum_{r_1=1}^{j_2} \dots \sum_{\substack{r_{j_2+1}=1 \\ (r_1+\dots+r_{j_2+1}=n)}} \frac{(n-1)!}{(r_1-1)! \dots (r_{j_2+1}-1)!} a_{r_1, r_1} \dots a_{r_{j_2+1}, r_{j_2+1}} \prod_{j_1=1}^{j_2-1} \prod_{k_1=1}^{j_1} \frac{1}{r_{k_1+1}+\dots+r_{j_2+1 k_2=j_1+1}} \\ & \frac{1}{r_{k_2+1}+\dots+r_{j_2+1}} + (-1)^2 \sum_{p_1=1}^{n-3} \left(\sum_{j_2=p_1+2}^{n-1} (-1)^{\frac{j_2}{2}} \sum_{r_1=1}^{j_2} \dots \sum_{\substack{r_{j_2-p_1+1}=1 \\ (r_1+r_2+\dots+r_{j_2-p_1+1}=n-p_1)}} \frac{(n-1)!}{(r_1-1)! \dots (r_{j_2-p_1+1}-1)!} a_{r_1, r_1} \dots a_{r_{j_2-p_1+1}, r_{j_2-p_1+1}} \prod_{j_1=p_1}^{j_2-1} \alpha_{p_1}^{1,1} \right. \\ & \left. \prod_{k_2=j_1-p_1+1}^{j_2-p_1} \frac{1}{r_{k_2+1}+\dots+r_{j_2-p_1+1}} \right) = -(-1)^{\frac{n-1}{2}} \frac{(n-1)!}{(n-1)!} + (-1)^2 \sum_{p_1=0}^{n-3} \left(\sum_{j_2=q_1}^{n-1} (-1)^{\frac{j_2}{2}} \right. \\ & \left. \sum_{r_1=1}^{j_2-1} \sum_{r_2=1}^{j_2-1} \dots \sum_{\substack{r_{j_2-p_1+1}=1 \\ (r_1+r_2+\dots+r_{j_2-p_1+1}=n-p_1)}} \frac{(n-1)!}{(r_1-1)! \dots (r_{j_2-p_1+1}-1)!} a_{r_1, r_1} \dots a_{r_{j_2-p_1+1}, r_{j_2-p_1+1}} \prod_{j_1=q_1}^{j_2-1} \alpha_{p_1}^{1,1} \right. \\ & \left. \prod_{k_2=j_1-p_1+1}^{j_2-p_1} \frac{1}{r_{k_2+1}+\dots+r_{j_2-p_1+1}} \right), \end{aligned} \quad (26)$$

wo $q_1 = \text{Max. } (p_1 + 2, 4)$ und $q_1' = \text{Max. } (p_1, 1)$ und

$$\alpha_{p_1}^{1,1} = \begin{cases} \frac{1}{p_1!} & (\text{wenn } j_1 = p_1 \text{ ist}) \\ \frac{1}{p_1!} \prod_{k_1=1}^{j_1-p_1} \frac{1}{p_1 + r_{k_1+1} + \dots + r_{j_2-p_1+1}} & (\text{wenn } j_1 > p_1 \text{ ist}) \end{cases}$$

Wenn in der Formel (26) $j_2 = n-1$ ist, so müssen alle $r_1, r_2, \dots, r_{j_2-p_1+1}$ gleich 1 sein, da $r_1 + \dots + r_{j_2-p_1+1} = r_1 + r_2 + \dots + r_{n-p_1} = n - p_1$ ist. Es ist aber

$$\begin{aligned} & \prod_{k_1=1}^{j_1-p_1} \frac{1}{p_1 + r_{k_1+1} + \dots + r_{j_2-p_1+1}} \prod_{k_2=j_1-p_1+1}^{j_2-p_1} \frac{1}{r_{k_2+1} + \dots + r_{j_2-p_1+1}} \\ &= \frac{(p_1 + j_2 - j_1)!}{j_2!} \frac{1}{(j_2 - j_1)!} \\ &= \frac{(j_2 - j_1 + p_1)(j_2 - j_1 + p_1 - 1) \dots (j_2 - j_1 + 1)}{j_2!}, \text{ wenn } j_1 > p_1 \text{ ist.} \quad (27) \end{aligned}$$

Daher ist

$$\begin{aligned} & \frac{1}{p_1!} \prod_{k_1=1}^{j_1-p_1} \frac{1}{p_1 + r_{k_1+1} + \dots + r_{j_2-p_1+1}} \prod_{k_2=j_1-p_1+1}^{j_2-p_1} \frac{1}{r_{k_2+1} + \dots + r_{j_2-p_1+1}} \\ &= \frac{1}{j_2!} j_2 - j_1 + p_1 C_{p_1}. \end{aligned}$$

Wenn $j_1 = p_1$ ist, ist es

$$\frac{1}{p_1!} \prod_{k_2=j_1-p_1+1}^{j_2-p_1} \frac{1}{r_{k_2+1} + \dots + r_{j_2-p_1+1}} = \frac{1}{p_1!} \frac{1}{(j_2 - p_1)!} = \frac{1}{j_2!} j_2 C_{p_1}. \quad (28)$$

Aus (26), (27) und (28) folgt es sofort

$$\begin{aligned} a_{n,n} &= -(-1)^{\frac{n-1}{2}} \frac{(n-1)!}{(n-1)!} + (-1)^2 (-1)^{\frac{n-1}{2}} \sum_{p_1=0}^{n-3} \sum_{j_1=q_1'}^{n-2} (n-1)! \beta_{p_1}^{1,1} \beta_{p_1}^{1,2} + (-1)^2 \\ & \sum_{p_1=0}^{n-3} \left(\sum_{j_2=q_1}^{n-2} (-1)^{\frac{j_2}{2}} \sum_{r_1=1}^{j_2-1} \dots \sum_{\substack{r_{j_2-p_1+1}=1 \\ (r_1+\dots+r_{j_2-p_1+1}=n-p_1)}} \frac{(n-1)!}{(r_1-1)! \dots (r_{j_2-p_1+1}-1)!} a_{r_1, r_1} \dots \right. \\ & a_{r_{j_2-p_1+1}, r_{j_2-p_1+1}} \sum_{j_1=q_1}^{j_2-1} \alpha_{p_1}^{1,1} \prod_{k_2=j_1-p_1+1}^{j_2-p_1} \frac{1}{r_{k_2+1} + \dots + r_{j_2-p_1+1}} = -(-1)^{\frac{n-1}{2}} \frac{(n-1)!}{(n-1)!} \\ & + (-1)^2 (-1)^{\frac{n-1}{2}} \sum_{p_1=0}^{n-3} \sum_{j_1=q_1}^{n-2} n-1-j_1+p_1 C_{p_1} + (-1)^2 \sum_{p_1=0}^{n-3} \left(\sum_{j_2=q_1}^{n-2} (-1)^{\frac{j_2}{2}} \sum_{r_1=1}^{j_2-1} \dots \sum_{\substack{r_{j_2-p_1+1}=1 \\ (r_1+\dots+r_{j_2-p_1+1}=n-p_1)}} \right. \\ & \left. \frac{(n-1)!}{(r_1-1)! \dots (r_{j_2-p_1+1}-1)!} a_{r_1, r_1} \dots a_{r_{j_2-p_1+1}, r_{j_2-p_1+1}} \sum_{j_1=q_1}^{j_2-1} \alpha_{p_1}^{1,1} \right) \end{aligned}$$

$$\prod_{k_2=j_1-p_1+1}^{j_2-p_1} \frac{1}{r_{k_2+1} + \dots + r_{j_2-p_1+1}}, \quad (29)$$

wo

$$\beta_{p_1}^{1,1} = \begin{cases} \frac{1}{p_1!} & (\text{wenn } j_1 = p_1 \text{ ist}) \\ \frac{1}{p_1!} \frac{(n-1-j_1+p_1)!}{(n-1)!} & (\text{wenn } j_1 > p_1 \text{ ist}) \end{cases} \quad \text{und } \beta_{p_1}^{1,2} = \frac{1}{(n-1-j_1)!} \text{ ist.}$$

Wenn in der Formel (26) $j_2 < n-1$ ist, so muss mindestens eine unter $r_1, r_2, \dots, r_{j_2-p_1+1}$ grösser als 1 sein. Wir nehmen erstens an, dass in der Formel (29) $r_{j_2-p_1+1} > 1$ ist. Es ist dann

$$\begin{aligned} & (-1)^{\frac{j_2}{2}} \sum_{r_1=1}^{j_2} \dots \sum_{r_{j_2-p_1+1} > 1} \frac{(n-1)!}{(r_1-1)! \dots (r_{j_2-p_1+1}-1)!} a_{r_1, r_1} \dots \\ & \quad (r_1 + \dots + r_{j_2-p_1+1} = n-p_1, r_{j_2-p_1+1} > 1) \\ & a_{r_{j_2-p_1+1}, r_{j_2-p_1+1}} \sum_{j_1=q_1}^{j_2-1} \alpha_{p_1}^{1,1} \prod_{k_2=j_1-p_1+1}^{j_2-p_1} \frac{1}{r_{k_2+1} + \dots + r_{j_2-p_1+1}} = (-1)^{\frac{j_2}{2}} \\ & \sum_{r_1=1}^{j_2} \dots \sum_{r_{j_2-p_1+1}=1} \frac{(n-1)!}{(r_1-1)! \dots (r_{j_2-p_1+1}-1)!} a_{r_1, r_1} \dots a_{r_{j_2-p_1+1}, r_{j_2-p_1+1}} \times (-1) \\ & \quad (r_1 + \dots + r_{j_2-p_1+1} = n-p_1-r_{j_2-p_1+1}) \\ & \sum_{j_3=1}^{r_{j_2-p_1+1}-1} (-1)^{\frac{j_3}{2}} \sum_{s_1=1}^{j_3} \dots \sum_{s_{j_3+1}=1} \frac{1}{(s_1-1)! \dots (s_{j_3+1}-1)!} a_{s_1, s_1} \dots a_{s_{j_3+1}, s_{j_3+1}} \\ & \quad (s_1 + s_2 + \dots + s_{j_3+1} = r_{j_2-p_1+1}) \\ & \prod_{k_3=1}^{j_3} \frac{1}{r_{j_2-p_1+1} - s_1 - \dots - s_{k_3}} \sum_{j_1=q_1}^{j_2-1} \alpha_{p_1}^{1,1} \prod_{k_2=j_1-p_1+1}^{j_2-p_1} \frac{1}{r_{k_2+1} + \dots + r_{j_2-p_1+1}}. \end{aligned}$$

Daraus ergibt es sich ohne weiteres

$$\begin{aligned} & \sum_{j_2=q_1}^{n-2} (-1)^{\frac{j_2}{2}} \sum_{r_1=1}^{j_2} \dots \sum_{r_{j_2-p_1+1} > 1} \frac{(n-1)!}{(r_1-1)! \dots (r_{j_2-p_1+1}-1)!} a_{r_1, r_1} \dots \\ & \quad (r_1 + \dots + r_{j_2-p_1+1} = n-p_1, r_{j_2-p_1+1} > 1) \\ & a_{r_{j_2-p_1+1}, r_{j_2-p_1+1}} \sum_{j_1=q_1}^{j_2-1} \alpha_{p_1}^{1,1} \prod_{k_2=j_1-p_1+1}^{j_2-p_1} \frac{1}{r_{k_2+1} + \dots + r_{j_2-p_1+1}} = - \sum_{j_3=q_1+2}^{n-1} (-1)^{\frac{j_3}{2}} \\ & \sum_{r_1=1}^{j_2} \dots \sum_{r_{j_3-p_1+1}=1} \frac{(n-1)!}{(r_1-1)! \dots (r_{j_3-p_1+1}-1)!} a_{r_1, r_1} \dots a_{r_{j_3-p_1+1}, r_{j_3-p_1+1}} \sum_{j_2=q_1}^{j_3-1} \sum_{j_1=q_1}^{j_2-1} \\ & \quad (r_1 + \dots + r_{j_3-p_1+1} = n-p_1) \\ & \alpha_{p_1, 0}^{2,1} \prod_{k_2=j_1-p_1+1}^{j_2-p_1} \frac{1}{r_{k_2+1} + \dots + r_{j_3-p_1+1}} \prod_{k_3=j_2-p_1+1}^{j_3-p_1} \frac{1}{r_{k_3+1} + \dots + r_{j_3-p_1+1}}, \quad (30) \end{aligned}$$

$$\text{wo } \alpha_{p_1,0}^{2,1} = \begin{cases} \frac{1}{p_1!} & (\text{wenn } j_1 = p_1 \text{ ist}) \\ \frac{1}{p_1!} \prod_{k_1=1}^{j_1-p_1} \frac{1}{p_1 + r_{k_1+1} + \dots + r_{j_3-p_1+1}} & (\text{wenn } j_1 > p_1 \text{ ist}) \end{cases} \text{ ist.}$$

Wir nehmen zweitens an, dass in der Formel (29) $r_{j_2-p_1} > 1$ und $r_{j_2-p_1+1} = 1$ ist. Es ist dann

$$\begin{aligned} & (-1)^{\frac{j_2}{2}} \sum_{r_1=1}^{j_2} \dots \sum_{\substack{r_{j_2-p_1} > 1, r_{j_2-p_1+1}=1 \\ (r_1+\dots+r_{j_2-p_1+1}=n-p_1, r_{j_2-p_1} > 1, r_{j_2-p_1+1}=1)}} \sum_{r_{j_2-p_1+1}=1}^{r_{j_2-p_1} > 1} \frac{(n-1)!}{(r_1-1)! \dots (r_{j_2-p_1+1}-1)!} a_{r_1, r_1} \dots \\ & a_{r_{j_2-p_1+1}, r_{j_2-p_1+1}} \sum_{j_1=q_1}^{j_2-1} \alpha_{p_1}^{1,1} \prod_{k_2=j_1-p_1+1}^{j_2-p_1} \frac{1}{r_{k_2+1} + \dots + r_{j_2-p_1+1}} = (-1)^{\frac{j_2}{2}} \\ & \sum_{r_1=1}^{j_2-1} \dots \sum_{\substack{r_{j_2-p_1-1}=1 \\ (r_1+\dots+r_{j_2-p_1-1}=n-p_1-1-r_{j_2-p_1})}} \frac{(n-1)!}{(r_1-1)! \dots (r_{j_2-p_1-1}-1)!} a_{r_1, r_1} \dots a_{r_{j_2-p_1-1}, r_{j_2-p_1-1}} \times (-1) \\ & \sum_{s_1=1}^{r_{j_2-p_1-1}} (-1)^{\frac{j_3}{2}} \sum_{s_1=1}^{j_3} \dots \sum_{\substack{s_{j_3+1}=1 \\ (s_1+\dots+s_{j_3+1}=r_{j_2-p_1})}} \frac{1}{(s_1-1)! \dots (s_{j_3+1}-1)!} a_{s_1, s_1} \dots a_{s_{j_3+1}, s_{j_3+1}} \prod_{k_3=1}^{j_3} \\ & \frac{1}{r_{j_2-p_1} - s_1 - \dots - s_{k_3}} \sum_{j_1=q_1}^{j_2-1} \alpha_{p_1}^{1,1} \prod_{k_2=j_1-p_1+1}^{j_2-p_1} \frac{1}{r_{k_2+1} + \dots + r_{j_2-p_1+1}}. \end{aligned}$$

Daher ist es

$$\begin{aligned} & \sum_{j_2=q_1}^{n-2} (-1)^{\frac{j_2}{2}} \sum_{r_1=1}^{j_2} \dots \sum_{\substack{r_{j_2-p_1} > 1, r_{j_2-p_1+1}=1 \\ (r_1+\dots+r_{j_2-p_1+1}=n-p_1, r_{j_2-p_1} > 1, r_{j_2-p_1+1}=1)}} \sum_{r_{j_2-p_1+1}=1}^{r_{j_2-p_1} > 1} \frac{(n-1)!}{(r_1-1)! \dots (r_{j_2-p_1+1}-1)!} a_{r_1, r_1} \dots \\ & a_{r_{j_2-p_1+1}, r_{j_2-p_1+1}} \sum_{j_1=q_1}^{j_2-1} \alpha_{p_1}^{1,1} \prod_{k_2=j_1-p_1+1}^{j_2-p_1} \frac{1}{r_{k_2+1} + \dots + r_{j_2-p_1+1}} = - \sum_{j_3=q_1+2}^{n-1} (-1)^{\frac{j_3}{2}} \\ & \sum_{r_1=1}^{j_3-1} \sum_{r_2=1}^{j_3-1} \dots \sum_{\substack{r_{j_3-p_1}=1 \\ (r_1+r_2+\dots+r_{j_3-p_1}=n-p_1-1)}} \frac{(n-1)!}{(r_1-1)! \dots (r_{j_3-p_1}-1)!} a_{r_1, r_1} \dots a_{r_{j_3-p_1}, r_{j_3-p_1}} \sum_{j_2=q_1}^{j_3-1} \sum_{j_1=q_1}^{j_2-1} \\ & \alpha_{p_1,1}^{2,1} \alpha_{p_1,1}^{2,2} \prod_{k_3=j_2-p_1}^{j_3-p_1-1} \frac{1}{r_{k_3+1} + \dots + r_{j_3-p_1}}, \quad (31) \\ & \alpha_{p_1,1}^{2,1} = \begin{cases} \frac{1}{p_1!} & (\text{wenn } j_1 = p_1 \text{ ist}) \\ \frac{1}{p_1!} \prod_{k_1=1}^{j_1-p_1} \frac{1}{p_1+1+r_{k_1+1}+\dots+r_{j_3-p_1}} & (\text{wenn } j_1 > p_1 \text{ ist}) \end{cases} \text{ und} \\ & \alpha_{p_1,1}^{2,2} = \prod_{k_2=j_1-p_1+1}^{j_2-p_1-1} \frac{1}{1+r_{k_2+1}+\dots+r_{j_3-p_1}} \text{ ist.} \end{aligned}$$

Dieses Verfahren setzen wir fort, so gewinnen wir

$$\begin{aligned}
& \sum_{j_2=q_1}^{n-2} (-1)^{\frac{j_2}{2}} \sum_{r_1=1}^{j_2} \cdots \sum_{r_{j_2-p_1+1}=1}^{j_2-p_1+1} \frac{(n-1)!}{(r_1-1)! \cdots (r_{j_2-p_1+1}-1)!} a_{r_1, r_1} \cdots \\
& \quad (r_1 + \cdots + r_{j_2-p_1+1} = n-1) \\
& a_{r_{j_2-p_1+1}, r_{j_2-p_1+1}} \sum_{j_1=q_1}^{j_2-1} \alpha_{p_1}^{1,1} \prod_{k_2=j_1-p_1+1}^{j_2-p_1} \frac{1}{r_{k_2+1} + \cdots + r_{j_2-p_1+1}} \\
& = - \sum_{p_2=0}^{n-3} \sum_{j_3=q_2}^{n-1} (-1)^{\frac{j_3}{2}} \sum_{r_1=1}^{j_3} \cdots \sum_{r_{j_3-p_1-p_2+1}=1}^{j_3-p_1-p_2+1} \frac{(n-1)!}{(r_1-1)! \cdots (r_{j_3-p_1-p_2+1}-1)!} a_{r_1, r_1} \cdots \\
& \quad (r_1 + \cdots + r_{j_3-p_1-p_2+1} = n-p_1-p_2) \\
& a_{r_{j_3-p_1-p_2+1}, r_{j_3-p_1-p_2+1}} \sum_{j_2=q_2}^{j_3-1} \sum_{j_1=q_1}^{j_2-1} \alpha_{p_1, p_2}^{2,1} \alpha_{p_1, p_2}^{2,2} \alpha_{p_1, p_2}^{2,3}, \quad (32)
\end{aligned}$$

wo

$$\begin{aligned}
\alpha_{p_1, p_2}^{2,1} &= \begin{cases} \frac{1}{p_1!} & \text{(wenn } j_1 = p_1 \text{ ist)} \\ \frac{1}{p_1!} \prod_{k_1=1}^{j_1-p_1} \frac{1}{p_1 + p_2 + r_{k_1+1} + \cdots + r_{j_3-p_1-p_2+1}} & \text{(wenn } j_1 > p_1 \text{ und } j_1 - p_1 + 1 \leq j_2 - p_1 - p_2 + 1 \text{ ist)} \\ \frac{1}{p_1!} \frac{(p_1 + j_2 - j_1)!}{(p_1 + p_2)!} \prod_{k_1=1}^{j_2-p_1-p_2} \frac{1}{p_1 + p_2 + r_{k_1+1} + \cdots + r_{j_3-p_1-p_2+1}} & \text{(wenn } j_1 > p_1 \text{ und } j_1 - p_1 + 1 > j_2 - p_1 - p_2 + 1 \text{ ist)} \end{cases} \quad \text{und} \\
\alpha_{p_1, p_2}^{2,2} &= \begin{cases} \frac{1}{(j_2 - j_1)!} & \text{(wenn } j_2 - p_1 - p_2 + 1 \leq j_1 - p_1 + 1 \text{ ist)} \\ \frac{1}{p_2!} \prod_{k_2=j_1-p_1+1}^{j_2-p_1-p_2} \frac{1}{p_2 + r_{k_2+1} + \cdots + r_{j_3-p_1-p_2+1}} & \text{(wenn } j_2 - p_1 - p_2 + 1 > j_1 - p_1 + 1 \text{ ist)} \end{cases} \quad \text{und} \\
\alpha_{p_1, p_2}^{2,3} &= \prod_{k_3=j_2-p_1-p_2+1}^{j_3-p_1-p_2} \frac{1}{r_{k_3+1} + \cdots + r_{j_3-p_1-p_2+1}} \quad \text{und}
\end{aligned}$$

$q_2 = \text{Max.}(4, p_1 + 4, p_1 + p_2 + 2)$ und

$q_2' = \text{Max.}(4, p_1 + 2, p_1 + p_2)$ ist.

Aus (32) und (29) folgt es

$$\begin{aligned}
a_{n,n} &= -(-1)^{\frac{n-1}{2}} \frac{(n-1)!}{(n-1)!} + (-1)^2 (-1)^{\frac{n-1}{2}} \sum_{p_1=0}^{n-3} \sum_{j_1=q_1}^{n-2} (n-1)! \beta_{p_1}^{1,1} \beta_{p_1}^{1,2} + (-1)^3 \\
& \sum_{p_1=0}^{n-3} \sum_{p_2=0}^{n-3} \sum_{j_3=q_2}^{n-1} (-1)^{\frac{j_3}{2}} \sum_{r_1=1}^{j_3} \sum_{r_2=1}^{j_3} \cdots \sum_{r_{j_3-p_1-p_2+1}=1}^{j_3-p_1-p_2+1} \frac{(n-1)!}{(r_1-1)! \cdots (r_{j_3-p_1-p_2+1}-1)!} a_{r_1, r_1} \cdots \\
& \quad (r_1 + \cdots + r_{j_3-p_1-p_2+1} = n-p_1-p_2) \\
& a_{r_{j_3-p_1-p_2+1}, r_{j_3-p_1-p_2+1}} \sum_{j_2=q_2}^{j_3-1} \sum_{j_1=q_1}^{j_2-1} \alpha_{p_1, p_2}^{2,1} \alpha_{p_1, p_2}^{2,2} \alpha_{p_1, p_2}^{2,3}. \quad (33)
\end{aligned}$$

Wenn $j_3 = n-1$ ist, so müssen alle $r_1, r_2, \dots, r_{j_3-p_1-p_2+1}$ gleich 1 sein,

da $r_1 + r_2 + \cdots + r_{j_3-p_1-p_2+1} = n-p_1-p_2$ ist. Daher ist für $j_3 = n-1$

$$\alpha_{p_1, p_2}^{2,1} = \beta_{p_1, p_2}^{2,1} = \begin{cases} \frac{1}{p_1!} & (\text{wenn } j_1 = p_1 \text{ ist}) \\ \frac{1}{p_1!} \frac{(p_1 + j_3 - j_1)!}{(n-1)!} & (\text{wenn } j_1 > p_1 \text{ und } j_1 - p_1 + 1 \leq j_2 - p_1 - p_2 + 1 \text{ ist}) \\ \frac{1}{p_1!} \frac{(p_1 + j_2 - j_1)!}{(p_1 + p_2)!} \frac{(j_3 - j_2 + p_1 + p_2)!}{(n-1)!} & (\text{wenn } j_1 > p_1 \text{ und } j_1 - p_1 + 1 > j_2 - p_1 - p_2 + 1 \text{ ist}) \end{cases}$$

$$\alpha_{p_1, p_2}^{2,2} = \beta_{p_1, p_2}^{2,2} = \begin{cases} \frac{1}{(j_3 - j_1)!} & (\text{wenn } j_2 - p_1 - p_2 + 1 \leq j_1 - p_1 + 1 \text{ ist}) \\ \frac{1}{p_2!} \frac{(p_2 + j_3 - j_2)!}{(n-1-j_1)!} & (\text{wenn } j_2 - p_1 - p_2 + 1 > j_1 - p_1 + 1 \text{ ist}) \end{cases}$$

$$\alpha_{p_1, p_2}^{2,3} = \beta_{p_1, p_2}^{2,3} = \frac{1}{(j_3 - j_2)!} \quad (34)$$

Aus (33) und (34) folgt es ohne weiteres

$$a_{n,n} = -(-1)^{\frac{n-1}{2}} \frac{(n-1)!}{(n-1)!} + (-1)^2 (-1)^{\frac{n-1}{2}} \sum_{p_1=0}^{n-3} \sum_{j_1=q_1'}^{n-2} (n-1)! \beta_{p_1}^{1,1} \beta_{p_1}^{1,2} \\ + (-1)^3 (-1)^{\frac{n-1}{2}} \sum_{p_1=0}^{n-3} \sum_{p_2=0}^{n-3} \sum_{j_2=q_2'}^{n-2} \sum_{j_1=q_1'}^{j_2-1} (n-1)! \beta_{p_1, p_2}^{2,1} \beta_{p_1, p_2}^{2,2} \beta_{p_1, p_2}^{2,3} + (-1)^3 \\ \sum_{p_1=0}^{n-3} \sum_{p_2=0}^{n-3} \sum_{j_3=q_3}^{n-2} (-1)^{\frac{j_3}{2}} \sum_{r_1=1}^{j_3} \sum_{r_2=1}^{j_3} \dots \sum_{r_{j_3-p_1-p_2+1}=1}^{j_3} \frac{(n-1)!}{(r_1-1)! \dots (r_{j_3-p_1-p_2+1}-1)!} a_{r_1, r_1, \dots} \\ (r_1+r_2+\dots+r_{j_3-p_1-p_2+1}=n-p_1-p_2) \\ a_{r_{j_3-p_1-p_2+1}, r_{j_3-p_1-p_2+1}} \sum_{j_2=q_2'}^{j_3-1} \sum_{j_1=q_1'}^{j_2-1} \alpha_{p_1, p_2}^{2,1} \alpha_{p_1, p_2}^{2,2} \alpha_{p_1, p_2}^{2,3}.$$

Dieses Verfahren setzen wir fort, so bekommen wir

$$a_{n,n} = -(-1)^{\frac{n-1}{2}} \frac{(n-1)!}{(n-1)!} + (-1)^2 (-1)^{\frac{n-1}{2}} \sum_{p_1=0}^{n-3} \sum_{j_1=q_1'}^{n-2} (n-1)! \beta_{p_1}^{1,1} \beta_{p_1}^{1,2} + \dots \\ + (-1)^{s+1} (-1)^{\frac{n-1}{2}} \sum_{p_1=0}^{n-3} \sum_{p_2=0}^{n-3} \dots \sum_{p_s=0}^{n-3} \sum_{j_s=q_s'}^{n-2} \sum_{j_{s-1}=q_{s-1}'}^{j_s-1} \dots \sum_{j_1=q_1'}^{j_2-1} (n-1)! \beta_{p_1, p_2, \dots, p_s}^{s,1} \\ \beta_{p_1, p_2, \dots, p_s}^{s,2} \dots \beta_{p_1, p_2, \dots, p_s}^{s, s+1} + (-1)^{s+1} \sum_{p_1=0}^{n-3} \sum_{p_2=0}^{n-3} \dots \sum_{p_s=0}^{n-3} \sum_{j_{s+1}=q_{s+1}}^{n-2} (-1)^{\frac{j_{s+1}}{2}} \sum_{r_1=1}^{j_{s+1}} \sum_{r_2=1}^{j_{s+1}} \dots \\ \sum_{r_{j_{s+1}-p_1-\dots-p_{s+1}}=1}^{j_{s+1}} \frac{(n-1)!}{(r_1-1)! \dots (r_{j_{s+1}-p_1-\dots-p_{s+1}}-1)!} a_{r_1, r_1, \dots} \\ (r_1+r_2+\dots+r_{j_{s+1}-p_1-\dots-p_{s+1}}=n-p_1-\dots-p_s) \\ a_{r_{j_{s+1}-p_1-\dots-p_{s+1}}, r_{j_{s+1}-p_1-\dots-p_{s+1}}} \sum_{j_s=q_s'}^{j_{s+1}-1} \sum_{j_{s-1}=q_{s-1}'}^{j_s-1} \dots \sum_{j_1=q_1'}^{j_2-1} \alpha_{p_1, p_2, \dots, p_s}^{s,1} \alpha_{p_1, p_2, \dots, p_s}^{s,2} \dots \alpha_{p_1, p_2, \dots, p_s}^{s, s+1} \quad (35)$$

wo $q'_s = \text{Max. } (q_{s-1}, p_1 + p_2 + \dots + p_s)$ und $q_s = \text{Max. } (2s, p_1 + 2 + 2(s-1), p_1 + p_2 + 2(s-2), \dots, p_1 + \dots + p_{s-1} + 4, p_1 + p_2 + \dots + p_s + 2)$ ist und $\alpha_{p_1, p_2, \dots, p_s}^{s,j}$ und $\beta_{p_1, p_2, \dots, p_s}^{s,j}$ die folgenden Sinne haben,

Für die Zahlen $j_1 - p_1 + 1, j_2 - p_1 - p_2 + 1, \dots, j_s - p_1 - p_2 - \dots - p_s + 1$ definieren wir eine Ordnung. Zwei Zahlen $j_i - p_1 - \dots - p_i + 1$ und $j_l - p_1 - \dots - p_l + 1$ heißen wohl geordnet, wenn $j_i - p_1 - \dots - p_i + 1 \leq j_l - p_1 - \dots - p_l + 1$ für $i < l$ ist. Wenn $j_i - p_1 - \dots - p_i + 1 > j_l - p_1 - \dots - p_l + 1$ für $i < l$ ist, so heißen $j_i - p_1 - \dots - p_i + 1$ und $j_l - p_1 - \dots - p_l + 1$ umgekehrt geordnet.

Wenn $j_1 - p_1 + 1$ und $j_2 - p_1 - p_2 + 1$ wohl geordnet sind, so setzen wir $f_1^1 = 1$. Wenn $j_1 - p_1 + 1$ und $j_2 - p_1 - p_2 + 1$ umgekehrt geordnet sind, so setzen wir $f_1^1 = \frac{(p_1 + j_2 - j_1)!}{(p_1 + p_2)!}$.

Wenn $j_3 - p_1 - p_2 - p_3 + 1$ und das Minimum von $j_1 - p_1 + 1$ und $j_2 - p_1 - p_2 + 1$ wohl geordnet sind, so setzen wir $f_2^1 = 1$. Wenn $j_3 - p_1 - p_2 - p_3 + 1$ und das Minimum von $j_1 - p_1 + 1$ und $j_2 - p_1 - p_2 + 1$ umgekehrt geordnet sind, so setzen wir $f_2^1 = \frac{(p_1 + j_3 - j_1)!}{(p_1 + p_2 + p_3)!}$ oder $f_2^1 = \frac{(p_1 + p_2 + j_3 - j_2)!}{(p_1 + p_2 + p_3)!}$, je nachdem das Minimum von $j_1 - p_1 + 1$ und $j_2 - p_1 - p_2 + 1$ mit $j_1 - p_1 + 1$ oder mit $j_2 - p_1 - p_2 + 1$ übereinstimmt.

Im allgemeinen setzen wir $\text{Min.}(j_1 - p_1 + 1, j_2 - p_1 - p_2 + 1, \dots, j_{i-1} - p_1 - p_2 - \dots - p_{i-1} + 1) = j_i - p_1 - \dots - p_i + 1$. Wenn $j_i - p_1 - \dots - p_i + 1$ und $j_l - p_1 - \dots - p_l + 1$ wohl geordnet sind, so setzen wir $f_i^1 = 1$. Wenn $j_i - p_1 - \dots - p_i + 1$ und $j_l - p_1 - \dots - p_l + 1$ umgekehrt geordnet sind, so setzen wir $f_i^1 = \frac{(j_i - j_l + p_1 + p_2 + \dots + p_l)!}{(p_1 + p_2 + \dots + p_{i+1})!}$. Es besteht dann

$$\alpha_{p_1, p_2, \dots, p_s}^{s,1} = \begin{cases} \frac{1}{p_1!} & (\text{wenn } j_1 = p_1 \text{ ist}) \\ \frac{1}{p_1!} \frac{s-1}{\prod_{i=1}^{s-1} f_i^1} \frac{\delta_s^1}{\prod_{k_1=1}^{\delta_s^1}} \frac{1}{p_1 + p_2 + \dots + p_s + r_{k_1+1} + \dots + r_{j_{s+1}-p_1-\dots-p_s+1}} & (\text{wenn } j_1 > p_1 \text{ ist}) \end{cases} \quad (36)$$

wo $\delta_s^1 + 1 = \text{Min.}(j_1 - p_1 + 1, j_2 - p_1 - p_2 + 1, \dots, j_s - p_1 - \dots - p_s + 1)$ ist.

Ebenfalls setzen wir $\text{Min.}(j_i - p_1 - \dots - p_i + 1, \dots, j_{i-1} - p_1 - \dots - p_{i-1} + 1) = j_m - p_1 - \dots - p_m + 1$, wo $l-1 \geq i$ ist. Wenn $j_i - p_1 - \dots - p_i + 1$ und $j_m - p_1 - \dots - p_m + 1$ wohl geordnet sind, setzen wir $f_{i-1}^i = 1$. Wenn $j_i - p_1 - \dots - p_i + 1$ und $j_m - p_1 - \dots - p_m + 1$ umgekehrt geordnet sind, so setzen wir $f_{i-1}^i = \frac{(j_i - j_m + p_1 + p_{i+1} + \dots + p_m)!}{(p_i + p_{i+1} + \dots + p_l)!}$. Es besteht dann, für $i=2, \dots, s$,

$$\alpha_{p_1, p_2, \dots, p_s}^{s,i} = \begin{cases} \frac{1}{(j_i - j_{i-1})!} & (\text{wenn } j_i - p_1 - \dots - p_i + 1 \leq j_{i-1} - p_1 - \dots - p_{i-1} + 1 \text{ ist}). \\ \frac{1}{p_i!} \frac{s-1}{\prod_{j=i}^{s-1} f_j^i} \frac{\delta_i^s}{\prod_{k_i=j_{i-1}-p_1-\dots-p_{i-1}+1}^{\delta_i^s}} \frac{1}{p_i + p_{i+1} + \dots + p_s + r_{k_i+1} + \dots + r_{j_{s+1}-p_1-\dots-p_s+1}} \end{cases}$$

$$\left\{ \begin{array}{l} \text{(wenn jede von } j_i - p_1 - \dots - p_i + 1, j_{i+1} - p_1 - \dots - p_{i+1} + 1, \\ \dots, j_s - p_1 - p_2 - \dots - p_s + 1 \text{ grösser als } j_{i-1} - p_1 - \dots - p_{i-1} + 1 \\ \text{ist).} \\ \frac{1}{p_i!} \prod_{j=i}^{i-1} f_j \frac{(j_i - j_m + p_i + p_{i+1} + \dots + p_m)!}{(j_i - j_{i-1})!}, \text{ (wenn für } i \leq l \\ \leq s \text{ es } j_i - p_1 - \dots - p_i + 1 \leq j_{i-1} - p_1 - \dots - p_{i-1} + 1 \text{ besteht} \\ \text{und wenn jede von } j_i - p_1 - \dots - p_i + 1, j_{i+1} - p_1 - \dots - p_{i+1} \\ + 1, \dots, j_{i-1} - p_1 - \dots - p_{i-1} + 1 \text{ grösser als } j_{i-1} - p_1 - \dots - \\ p_{i-1} + 1 \text{ ist)} \end{array} \right. \quad (37)$$

wo $\delta_i + 1 = \text{Min. } (j_i - p_1 - \dots - p_i + 1, \dots, j_s - p_1 - \dots - p_s + 1)$ und $\text{Min. } (j_i - p_1 - \dots - p_i + 1, \dots, j_{i-1} - p_1 - \dots - p_{i-1} + 1) = j_m - p_1 - \dots - p_m + 1$ ist.

$$\alpha_{p_1, p_2, \dots, p_s}^{s, s+1} = \frac{j_{s+1} - p_1 - \dots - p_s}{\prod_{k_{s+1}=j_s - p_1 - \dots - p_s + 1} r_{k_{s+1}+1} + \dots + r_{j_{s+1} - p_1 - \dots - p_s + 1}} \cdot \quad (38)$$

$$\beta_{p_1, p_2, \dots, p_s}^{s, 1} = \begin{cases} \frac{1}{p_1!} & \text{(wenn } j_1 = p_1 \text{ ist)} \\ \frac{1}{p_1!} \prod_{j=1}^{s-1} f_j \frac{(n-1-j_m + p_1 + \dots + p_m)!}{(n-1-j_{i-1})!}, & \text{(wenn } j_1 > p_1 \text{ ist)} \end{cases} \quad (39)$$

wo $\text{Min. } (j_1 - p_1 + 1, j_2 - p_1 - p_2 + 1, \dots, j_s - p_1 - \dots - p_s + 1) = j_m - p_1 - \dots - p_m + 1$ ist.

$$\beta_{p_1, p_2, \dots, p_s}^{s, i} = \left\{ \begin{array}{l} \frac{1}{(j_i - j_{i-1})!} \text{ (wenn } j_i - p_1 - p_2 - \dots - p_i + 1 \leq j_{i-1} - p_1 - \dots - p_{i-1} + 1 \text{ ist)} \\ \frac{1}{p_i!} \prod_{j=i}^{s-1} f_j \frac{(n-1-j_m + p_i + p_{i+1} + \dots + p_m)!}{(n-1-j_{i-1})!} \text{ (wenn jede von} \\ j_i - p_1 - \dots - p_i + 1, j_{i+1} - p_1 - \dots - p_{i+1} + 1, \dots, j_s - p_1 - \dots - p_s + 1 \\ \text{grösser als } j_{i-1} - p_1 - \dots - p_{i-1} + 1 \text{ ist), wo Min. } (j_i - p_1 - \dots - p_i + 1, \dots, j_s - p_1 - \dots - p_s + 1) = j_m - p_1 - \dots - p_m + 1 \text{ ist,} \\ \frac{1}{p_i!} \prod_{j=i}^{i-1} f_j \frac{(j_i - j_m + p_i + \dots + p_m)!}{(j_i - j_{i-1})!}, \text{ (wenn für } i \leq l \leq s \\ \text{es } j_i - p_1 - \dots - p_i + 1 \leq j_{i-1} - p_1 - \dots - p_{i-1} + 1 \text{ besteht und} \\ \text{wenn jede von } j_i - p_1 - \dots - p_i + 1, j_{i+1} - p_1 - \dots - p_{i+1} + 1, \dots \\ j_{i-1} - p_1 - \dots - p_{i-1} + 1 \text{ grösser als } j_{i-1} - p_1 - p_2 - \dots - p_{i-1} + 1 \\ \text{ist),} \end{array} \right. \quad (40)$$

wo $\text{Min. } (j_i - p_1 - \dots - p_i + 1, \dots, j_{i-1} - p_1 - \dots - p_{i-1} + 1) = j_m - p_1 - \dots - p_m + 1$ ist. Diese Formel gilt für $i=2, 3, \dots, s$.

$$l_{p_1, p_2, \dots, p_s}^{s, s+1} = \frac{1}{(n-1-j_s)!} \quad (41)$$

Dieses Verfahren setzen wir fort, so gewinnen wir endlich

$$a_{n,n} = -(-1)^{\frac{n-1}{2}} \frac{(n-1)!}{(n-1)!} + \sum_{s=1}^{\frac{n-1}{2}} (-1)^{s+1} (-1)^{\frac{n-1}{2}} \sum_{p_1=0}^{n-3} \sum_{p_2=0}^{n-3} \dots \sum_{p_s=0}^{n-3} \sum_{j_s=q_s}^{n-2} \sum_{j_{s-1}=q_{s-1}}^{j_s-1} \dots \sum_{j_1=q_1}^{j_2-1} (n-1)! l_{p_1, p_2, \dots, p_s}^{s, 1} l_{p_1, p_2, \dots, p_s}^{s, 2} \dots l_{p_1, p_2, \dots, p_s}^{s, s+1} \quad (42)$$

Dabei wird $l_{p_1, p_2, \dots, p_s}^{s, i}$ durch (39), (40) und (41) gegeben.

Wir behandeln nun den Fall, wo das Suffix n von $a_{n,n}$ gerade ist. Nach (16) ist $a_{2,2} = -\Im(\kappa A i^2) / \Re(\kappa A) = \frac{\Im(\kappa A)}{\Re(\kappa A)} = 0$, da $\Im(\kappa A) = 0$ ist.

Wir nehmen an, dass $a_{p,p} = 0$ ist für jede gerade positive ganze Zahl p , wo $2 \leq p < n$ ist. In der Formel (19) ist es

$$r_1 + r_2 + \dots + r_{j+1} = n, j = \text{gerade Zahl.}$$

Daher ist mindestens eine unter r_1, r_2, \dots, r_{j+1} eine gerade Zahl. Folglich ist

$$a_{n,n} = 0. \quad (43)$$

Diese Formel gilt für alle geraden Zahlen n .

Wir wollen nun die Koeffizient $a_{n, n-1}$ von $\varphi^{(n)}(0) = \sum_{j=0}^n a_{n,j} \varphi'(0)^j$ berechnen. Es ergibt sich aus (14) und (15)

$$\begin{aligned} & \Im \kappa A \left\{ i a_{n, n-1} + \sum_{j=1}^{n-1} \left(\sum_{p_j=1}^{n-j} \sum_{p_{j-1}=p_j+1}^{n-j+1} \dots \sum_{p_1=p_2+1}^{n-1} C_{p_1} C_{p_1-1} C_{p_2} \dots C_{p_{j-1}-1} C_{p_j} i^{j+1} \right. \right. \\ & \left. \left. \sum_{k_1=0} \sum_{k_2=0} \dots \sum_{k_{j+1}=0} a_{n-p_1, k_1} a_{p_1-p_2, k_2} \dots a_{p_{j-1}-p_j, k_j} a_{p_j, k_{j+1}} \right) \right\} + n C_1 \Im B \kappa^2 \left\{ i 2 a_{n-1, n-1} + \right. \\ & \left. \sum_{j=1}^{n-2} \left(\sum_{p_j=1}^{n-j-1} \sum_{p_{j-1}=p_j+1}^{n-j} \dots \sum_{p_1=p_2+1}^{n-2} 2^{j+1} C_{p_1} C_{p_1-1} C_{p_2} \dots C_{p_{j-1}-1} C_{p_j} i^{j+1} a_{n-1-p_1, n-1-p_1} \right. \right. \\ & \left. \left. a_{p_1-p_2, p_1-p_2} \dots a_{p_{j-1}-p_j, p_{j-1}-p_j} a_{p_j, p_j} \right) \right\} = 0, \quad (\text{für } n \leq 3) \end{aligned}$$

wo $\sum_{k_1=0} \sum_{k_2=0} \dots \sum_{k_{j+1}=0}$ über alle Systeme von 0 und positiven ganzen Zahlen $(k_1 + k_2 + \dots + k_{j+1} = n-1)$

k_1, k_2, \dots, k_{j+1} zu erstrecken ist, derart dass $k_1 + k_2 + \dots + k_{j+1} = n-1$ ist.

Daraus folgt ohne weiteres

$$a_{n, n-1} = -\Im \kappa A \sum_{j=1}^{n-1} \left(\sum_{p_j=1}^{n-j} \sum_{p_{j-1}=p_j+1}^{n-j+1} \dots \sum_{p_1=p_2+1}^{n-1} C_{p_1} C_{p_1-1} C_{p_2} \dots C_{p_{j-1}-1} C_{p_j} i^{j+1} \right.$$

$$\sum_{k_1=0} \sum_{k_2=0} \cdots \sum_{k_{j+1}=0} a_{n-p_1, k_1} a_{p_1-p_2, k_2} \cdots a_{p_{j-1}-p_j, k_j} a_{p_j, k_{j+1}} / \Re(\kappa A) - {}_n C_1 \Im B \kappa^2$$

$$\left\{ 2i a_{n-1, n-1} + \sum_{j=1}^{n-2} \left(\sum_{p_j=1}^{n-j} \sum_{p_{j-1}=p_j+1}^{n-j+1} \cdots \sum_{p_1=p_2+1}^{n-2} 2^{j+1} {}_{n-2} C_{p_1 p_1-1} C_{p_2} \cdots C_{p_{j-1}-1} C_{p_j} i^{j+1} \right. \right.$$

$$\left. a_{n-1-p_1, n-1-p_1} a_{p_1-p_2, p_1-p_2} \cdots a_{p_{j-1}-p_j, p_{j-1}-p_j} a_{p_j, p_j} \right\} / \Re(A \kappa). \quad (44)$$

Es ist aber

$$- \Im \kappa A \sum_{j=1}^{n-1} \left(\sum_{p_j=1}^{n-j} \sum_{p_{j-1}=p_j+1}^{n-j+1} \cdots \sum_{p_1=p_2+1}^{n-1} {}_{n-1} C_{p_1 p_1-1} C_{p_2} \cdots C_{p_{j-1}-1} C_{p_j} i^{j+1} \right.$$

$$\left. \sum_{k_1=0} \sum_{k_2=0} \cdots \sum_{k_{j+1}=0} a_{n-p_1, k_1} a_{p_1-p_2, k_2} \cdots a_{p_{j-1}-p_j, k_j} a_{p_j, k_{j+1}} \right) / \Re(\kappa A)$$

$$= \sum_{j=1}^{n-1} (-1)^{\frac{j}{2}} \sum_{r_1=1}^j \sum_{r_2=1}^j \cdots \sum_{r_{j+1}=1}^j \frac{(n-1)!}{(r_1-1)! (r_2-1)! \cdots (r_{j+1}-1)!} \sum_{s_1=0}^{j+1} a_{r_1, r_1} \cdots$$

$$(r_1+r_2+\cdots+r_{j+1}=n)$$

$$a_{r_{s_1-1}, r_{s_1-1}} a_{r_{s_1}, r_{s_1}-1} a_{r_{s_1+1}, r_{s_1+1}} \cdots a_{r_{j+1}, r_{j+1}} \prod_{k=1}^j \frac{1}{n-r_1-\cdots-r_k}. \quad (45)$$

Ebenfalls ist es

$${}_n C_1 \Im B \kappa^2 \left\{ 2i a_{n-1, n-1} + \sum_{j=1}^{n-2} \left(\sum_{p_j=1}^{n-j-1} \sum_{p_{j-1}=p_j+1}^{n-j} \cdots \sum_{p_1=p_2+1}^{n-2} 2^{j+1} i^{j+1} {}_{n-2} C_{p_1 p_1-1} C_{p_2} \cdots \right. \right.$$

$$\left. C_{p_{j-1}-1} C_{p_j} a_{n-1-p_1, n-1-p_1} a_{p_1-p_2, p_1-p_2} \cdots a_{p_{j-1}-p_j, p_{j-1}-p_j} a_{p_j, p_j} \right\} / \Re(\kappa A)$$

$$= 2 {}_n C_1 \Re(B \kappa^2) a_{n-1, n-1} / \Re(\kappa A) + {}_n C_1 \sum_{j=1}^{n-2} \Im(B \kappa^2 i^{j+1}) \sum_{r_1=1}^j \sum_{r_2=1}^j \cdots \sum_{r_{j+1}=1}^j$$

$$(k_1+r_2+\cdots+r_{j+1}=n-1)$$

$$\frac{(n-2)! 2^{j+1}}{(r_1-1)! (r_2-1)! \cdots (r_{j+1}-1)!} a_{r_1, r_1} \cdots a_{r_{j+1}, r_{j+1}}$$

$$\prod_{k=1}^j \frac{1}{n-1-r_1-\cdots-r_k} / \Re(\kappa A). \quad (46)$$

Aus (44), (45) und (46) bekommen wir

$$a_{n, n-1} = - \sum_{j=1}^{n-1} (-1)^{\frac{j}{2}} \sum_{r_1=1}^j \cdots \sum_{r_{j+1}=1}^j \frac{(n-1)!}{(r_1-1)! (r_2-1)! \cdots (r_{j+1}-1)!}$$

$$(r_1+\cdots+r_{j+1}=n)$$

$$\sum_{s_1=0}^{j+1} a_{r_1, r_1} \cdots a_{r_{s_1}, r_{s_1}-1} \cdots a_{r_{j+1}, r_{j+1}} \prod_{k=1}^j \frac{1}{n-r_1-\cdots-r_k}$$

$$- 2 {}_n C_1 \Re(B \kappa^2) a_{n-1, n-1} / \Re(\kappa A) - {}_n C_1 \sum_{j=1}^{n-2} \Im(B \kappa^2 i^{j+1}) \sum_{r_1=1}^j \cdots \sum_{r_{j+1}=1}^j$$

$$(r_1+\cdots+r_{j+1}=n-1)$$

$$\frac{(n-2)! 2^{j+1}}{(r_1-1)! \cdots (r_{j+1}-1)!} a_{r_1, r_1} \cdots a_{r_{j+1}, r_{j+1}} \prod_{k=1}^j \frac{1}{n-1-r_1-\cdots-r_k} \Big/ \Re(\kappa A). \quad (\text{für } n \geq 3) \quad (47)$$

Wir wollen nun das erste Glied von der rechten Seite der obigen Formel berechnen. Es ist

$$\begin{aligned} & \sum_{\substack{r_1=1 \\ (r_1+\cdots+r_{j+1}=n)}} \cdots \sum_{r_{j+1}=1} \frac{(n-1)!}{(r_1-1)! \cdots (r_{j+1}-1)!} a_{r_1, r_1-1} a_{r_2, r_2} \cdots a_{r_{j+1}, r_{j+1}} \prod_{k=1}^j \frac{1}{n-r_1-\cdots-r_k} \\ &= \sum_{r_2=1} \sum_{r_3=1} \cdots \sum_{r_{j+1}=1} \frac{(n-1)!}{(r_2-1)! \cdots (r_{j+1}-1)!} a_{r_2, r_2} a_{r_3, r_3} \cdots \\ & \quad a_{r_{j+1}, r_{j+1}} \frac{1}{n-2} \prod_{k=2}^j \frac{1}{n-2-r_2-\cdots-r_k} a_{2,1} + \sum_{\substack{r_1=3 \\ (r_1+r_2+\cdots+r_{j+1}=n, r_1 \geq 3)}} \sum_{r_2=1} \cdots \sum_{r_{j+1}=1} \frac{(n-1)!}{(r_1-1)! (r_2-1)! \cdots (r_{j+1}-1)!} a_{r_1, r_1-1} a_{r_2, r_2} \cdots a_{r_{j+1}, r_{j+1}} \prod_{k=1}^j \frac{1}{n-r_1-\cdots-r_k}. \end{aligned}$$

Das zweite Glied von der rechten Seite der obigen Formel

$$\begin{aligned} &= \sum_{\substack{r_1=3 \\ (r_1+\cdots+r_{j+1}=n, r_1 \geq 3)}} \sum_{r_2=1} \cdots \sum_{r_{j+1}=1} \frac{(n-1)!}{(r_1-1)! \cdots (r_{j+1}-1)!} a_{r_2, r_2} a_{r_3, r_3} \cdots a_{r_{j+1}, r_{j+1}} \prod_{k=1}^j \frac{1}{n-r_1-\cdots-r_k} \\ & \quad \times \left[\sum_{j_2=1}^{r_1-1} (-1)^{\frac{j_2}{2}} \sum_{s_1=1} \cdots \sum_{s_{j_2+1}=1} \frac{(r_1-1)!}{(s_1-1)! \cdots (s_{j_2+1}-1)!} \right. \\ & \quad \left. \sum_{\substack{h_2=1 \\ (s_1+\cdots+s_{j_2+1}=r_1)}}^{j_2+1} a_{s_1, s_1} \cdots a_{s_{h_2}, s_{h_2}-1} \cdots a_{s_{j_2+1}, s_{j_2+1}} \prod_{k=2}^{j_2} \frac{1}{s_{k_2+1} + \cdots + s_{j_2+1}} \right]^{6)} \\ & \quad + 2r_1 C_1 \Re(B\kappa^2) a_{r_1-1, r_1-1} \Big/ \Re(\kappa A) + r_1 C_1 \sum_{j_2=1}^{r_1-2} \Im(B\kappa^2 i^{j_2+1}) \sum_{s_1=1} \cdots \sum_{\substack{s_{j_2+1}=1 \\ (s_1+\cdots+s_{j_2+1}=r_1-1)}} \frac{(r_1-2)! 2^{j_2+1}}{(s_1-1)! \cdots (s_{j_2+1}-1)!} a_{s_1, s_1} \cdots a_{s_{j_2+1}, s_{j_2+1}} \prod_{k=2}^{j_2} \frac{1}{s_{k_2+1} + \cdots + s_{j_2+1}} \Big/ \Re(\kappa A). \end{aligned}$$

Es ist aber

$$-\sum_{j=1}^{n-1} (-1)^{\frac{j}{2}} \sum_{r_1=3} \sum_{r_2=1} \cdots \sum_{r_{j+1}=1} \frac{(n-1)!}{(r_1-1)! \cdots (r_{j+1}-1)!} a_{r_2, r_2} \cdots$$

$(r_1+r_2+\cdots+r_{j+1}=n, r_1 \geq 3)$

⁶⁾ und ⁷⁾. Die Formeln 6) und 7) sind in der Tat voneinander verschieden, da in 6) $s_1+\cdots+s_{j_2+1}=r_1$ und in 7) $s_1+\cdots+s_{j_2+1}=r_1-1$ ist.

$$\begin{aligned}
 & a_{r_{j+1}, r_{j+1}} \times - \sum_{j_2=1}^{r_1-1} (-1)^{\frac{j_2}{2}} \sum_{s_1=1}^{j_2} \cdots \sum_{s_{j_2+1}=1}^{s_{j_2+1}-1} \frac{(r_1-1)}{(s_1-1)! \cdots (s_{j_2+1}-1)!} \sum_{h_2=1}^{j_2+1} a_{t_1, s_1 \dots} \\
 & \quad (s_1 + \cdots + s_{j_2+1} = r_1) \\
 & a_{s_{h_2}, s_{h_2}-1} \cdots a_{s_{j_2+1}, s_{j_2+1}} \prod_{k_2=1}^{j_2} \frac{1}{s_{k_2+1} + \cdots + s_{j_2+1}} \prod_{k_1=1}^{j_1} \frac{1}{n - r_1 - \cdots - r_{k_1}} \\
 & = (-1)^2 \sum_{j_2=2}^{n-1} (-1)^{\frac{j_2}{2}} \sum_{r_1=1}^{j_2} \cdots \sum_{r_{j_2+1}=1}^{(n-1)!} \frac{(n-1)!}{(r_1-1)! \cdots (r_{j_2+1}-1)!} \sum_{j_1=1}^{j_2-1} \sum_{h_2=1}^{j_2-j_1+1} a_{r_1, r_1 \dots} \\
 & \quad (r_1 + \cdots + r_{j_2+1} = n) \\
 & a_{r_{j_1+1}, r_{j_1+1}} \cdots a_{r_{j_1+h_2}, r_{j_1+h_2}-1} \cdots a_{r_{j_2+1}, r_{j_2+1}} \prod_{k_1=1}^{j_1} \frac{1}{r_{k_1} + \cdots + r_{j_1}} \\
 & \quad \prod_{k_2=1}^{j_2-j_1} \frac{1}{r_{j_1+k_2+1} + \cdots + r_{j_2+1}}. \tag{48}
 \end{aligned}$$

Es ist aber

$$\begin{aligned}
 & - \sum_{j=1}^{n-1} (-1)^{\frac{j}{2}} \sum_{r_1=3}^j \sum_{r_2=1}^{j_2} \cdots \sum_{r_{j+1}=1}^{(n-1)!} \frac{(n-1)!}{(r_1-1)! \cdots (r_{j+1}-1)!} \prod_{k_1=1}^j \frac{1}{n - r_1 - \cdots - r_{k_1}} \times \\
 & \quad (r_1 + \cdots + r_{j+1} = n, r_j \geq 3) \\
 & - 2r_1 C_1 \Re(B\kappa^2) a_{r_1-1, r_1-1} / \Re(\kappa A) = (-1)^3 \sum_{j_2=2}^{n-2} (-1)^{\frac{j_2}{2}} \sum_{r_1=1}^{j_2} \sum_{j_2=1}^{j_2-1} \cdots \sum_{r_{j_2+1}=1}^{(r_1+r_2+\cdots+r_{j_2+1}=n-1)} \\
 & \quad \frac{(n-1)!}{(r_1-1)! \cdots (r_{j_2+1}-1)!} a_{r_1, r_1} \cdots a_{r_{j_2+1}, r_{j_2+1}} \sum_{j_1=1}^{j_2-1} \frac{r_{j_1+1} + \cdots + r_{j_2+1} + 1}{r_{j_1+1} + \cdots + r_{j_2+1}} 2 \\
 & \quad \prod_{k_1=1}^{j_1} \frac{1}{r_{k_1} + \cdots + r_{j_1}} \prod_{k_2=1}^{j_2-j_1} \frac{1}{r_{j_1+k_2+1} + \cdots + r_{j_2+1}} \Re(B\kappa^3) / \Re(\kappa A). \tag{49}
 \end{aligned}$$

Es ist aber

$$\begin{aligned}
 & - \sum_{j=1}^{n-1} (-1)^{\frac{j}{2}} \sum_{r_1=3}^j \sum_{r_2=1}^{j_2} \cdots \sum_{r_{j+1}=1}^{(n-1)!} \frac{(n-1)!}{(r_1-1)! (r_2-1)! \cdots (r_{j+1}-1)!} a_{r_2, r_2} \cdots \\
 & \quad (r_1 + r_2 + \cdots + r_{j+1} = n, r_1 \geq 3) \\
 & a_{r_{j+1}, r_{j+1}} \times - r_1 C_1 \left[\sum_{j_2=1}^{r_1-2} (-1)^{\frac{j_2}{2}} \Re(B\kappa^2) + \sum_{j_2=1}^{r_1-2} (-1)^{\frac{j_2+1}{2}} \Im(B\kappa^2) \right] \sum_{s_1=1}^{j_2} \cdots \sum_{s_{j_2+1}=1}^{(s_1+\cdots+s_{j_2+1}=r_1-1)} \\
 & \quad \frac{(r_1-2)! 2^{j_2+1}}{(s_1-1)! \cdots (s_{j_2+1}-1)!} a_{s_1, s_1} \cdots a_{s_{j_2+1}, s_{j_2+1}} \prod_{k_2=1}^{j_2} \frac{1}{s_{k_2+1} + \cdots + s_{j_2+1}} \\
 & \quad \prod_{k_1=1}^j \frac{1}{n - r_1 - \cdots - r_{k_1}} / \Re(\kappa A) \\
 & = (-1)^2 \sum_{j_2=2}^{n-2} (-1)^{\frac{j_2}{2}} \sum_{r_1=1}^{j_2} \cdots \sum_{r_{j_2+1}=1}^{(n-1)!} \frac{(n-1)!}{(r_1-1)! \cdots (r_{j_2+1}-1)!} a_{r_1, r_1} \dots \\
 & \quad (r_1 + \cdots + r_{j_2+1} = n-1)
 \end{aligned}$$

$$\begin{aligned}
& a_{r_{j_2+1}, r_{j_2+1}} \sum_{j_1=1}^{j_2-1} \frac{r_{j_1+1} + \dots + r_{j_2+1} + 1}{r_{j_1+1} + \dots + r_{j_2+1}} 2^{j_2-j_1+1} \prod_{k_1=1}^{j_1} \frac{1}{r_{k_1} + \dots + r_{j_1}} \\
& \prod_{k_2=1}^{j_2-j_1} \frac{1}{r_{j_1+k_2+1} + \dots + r_{j_2+1}} \Re(B\kappa^2) / \Re(\kappa A) \\
& + (-1)^2 \sum_{j_2=2}^{n-2} (-1)^{\frac{j_2+1}{2}} \sum_{r_1=1} \dots \sum_{(r_1+\dots+r_{j_2+1}=n-1)} \frac{(n-1)!}{(r_1-1)! \dots (r_{j_2+1}-1)!} a_{r_1, r_1} \dots \\
& a_{r_{j_2+1}, r_{j_2+1}} \sum_{j_1=1}^{j_2-1} \frac{r_{j_1+1} + \dots + r_{j_2+1} + 1}{r_{j_1+1} + \dots + r_{j_2+1}} 2^{j_2-j_1+1} \prod_{k_1=1}^{j_1} \frac{1}{r_{k_1} + \dots + r_{j_1}} \\
& \prod_{k_2=1}^{j_2-j_1} \frac{1}{r_{j_1+k_2+1} + \dots + r_{j_2+1}} \Im(B\kappa^2) / \Re(\kappa A), \tag{50}
\end{aligned}$$

wo die Summe $\sum_{j_2=2}^{n-2}$ über alle positiven ungeraden Zahlen bis 2 zu $n-2$ zu erstrecken ist.

Ebenfalls ist es

$$\begin{aligned}
& \sum_{r_1=1} \dots \sum_{(r_1+\dots+r_{j+1}=n)} \frac{(n-1)!}{(r_1-1)! \dots (r_{j+1}-1)!} a_{r_1, r_1} a_{r_2, r_2-1} \dots a_{r_{j+1}, r_{j+1}} \\
& \prod_{k=1}^j \frac{1}{n-r_1-\dots-r_k} = \sum_{r_1=1} \dots \sum_{(r_1+\dots+r_{j+1}=n-2)} \frac{(n-1)!}{(r_1-1)! \dots (r_j-1)!} a_{r_1, r_1} a_{r_2, r_2} \dots \\
& a_{r_j, r_j} \frac{1}{n-r_1} \prod_{k=2}^j \frac{1}{r_k + \dots + r_j} a_{2,1} + \sum_{r_1=1} \sum_{r_2=3} \dots \sum_{\substack{(r_1+\dots+r_{j+1}=n, \\ r_2 \geq 3)}} \frac{(n-1)!}{(r_1-1)! \dots (r_{j+1}-1)!} a_{r_1, r_1} a_{r_2, r_2-1} \dots a_{r_{j+1}, r_{j+1}} \prod_{k=1}^j \frac{1}{n-r_1-\dots-r_k}.
\end{aligned}$$

Es ist nun

$$\begin{aligned}
& - \sum_{j=1}^{n-1} (-1)^{\frac{j}{2}} \sum_{r_1=1} \sum_{r_2=3} \dots \sum_{\substack{(r_1+\dots+r_{j+1}=n, \\ r_2 \geq 3)}} \frac{(n-1)!}{(r_1-1)! \dots (r_{j+1}-1)!} a_{r_1, r_1} a_{r_2, r_2-1} \dots \\
& a_{r_{j+1}, r_{j+1}} \prod_{k=1}^j \frac{1}{n-r_1-\dots-r_k} = - \sum_{j=1}^{n-1} (-1)^{\frac{j}{2}} \sum_{r_1=1} \sum_{r_2=3} \dots \sum_{\substack{(r_1+\dots+r_{j+1}=n, \\ r_2 \geq 3)}} \frac{(n-1)!}{(r_1-1)! \dots (r_{j+1}-1)!} a_{r_1, r_1} a_{r_3, r_3} \dots a_{r_{j+1}, r_{j+1}} \times \\
& \times - \left[\sum_{j_2=1}^{r_2-1} (-1)^{\frac{j_2}{2}} \sum_{s_1=1} \dots \sum_{\substack{(s_1+\dots+s_{j_2+1}=r_2)}} \frac{(r_2-1)!}{(s_1-1)! \dots (s_{j_2+1}-1)!} \sum_{h_2=1}^{j_2+1} a_{s_1, s_1} \dots a_{s_{h_2}, s_{h_2}-1} \dots a_{s_{j_2+1}, s_{j_2+1}} \right]
\end{aligned}$$

$$\begin{aligned}
 & \prod_{k_2=1}^{j_2} \frac{1}{s_{k_2+1} + \dots + s_{j_2+1}} + 2r_2 C_1 \Re(B\kappa^2) a_{r_2-1, r_2-1} / \Re(\kappa A) + r_2 C_1 \\
 & \left\{ \sum_{j_2=1}^{r_2-2} (-1)^{\frac{j_2}{2}} \Re(B\kappa^2) + \sum_{j_2=1}^{r_2-2} (-1)^{\frac{j_2+1}{2}} \Im(B\kappa^2) \right\} \sum_{s_1=1} \dots \sum_{s_{j_2+1}=1} \\
 & \quad (s_1 + \dots + s_{j_2+1} = r_2 - 1) \\
 & \frac{(r_2 - 2)! 2^{j_2+1}}{(s_1 - 1)! \dots (s_{j_2+1} - 1)!} a_{s_1, s_1} \dots a_{s_{j_2+1}, s_{j_2+1}} \prod_{k_2=1}^{j_2} \frac{1}{s_{k_2+1} + \dots + s_{j_2+1}} / \Re(\kappa A) \Big] \\
 & \prod_{k=1}^j \frac{1}{n - r_1 - \dots - r_k} \\
 & = (-1)^3 \sum_{j_2=2}^{n-1} (-1)^{\frac{j_2}{2}} \sum_{r_1=1} \dots \sum_{r_{j_2+1}=1} \frac{(n-1)!}{(r_1 - 1)! \dots (r_{j_2+1} - 1)!} \sum_{j_1=1}^{j_2-1} \sum_{h_2=1}^{j_2-j_1+1} a_{r_1, r_1} \dots \\
 & \quad (r_1 + \dots + r_{j_2+1} = n) \\
 & a_{r_{j_1+1}, r_{j_1+1}} \dots a_{r_{j_1+h_2}, r_{j_1+h_2}} a_{r_{j_2+1}, r_{j_2+1}} \frac{1}{n - r_1} \prod_{k_1=2}^{j_1} \frac{1}{r_{k_1} + \dots + r_{j_1}} \\
 & \prod_{k_2=1}^{j_2-j_1} \frac{1}{r_{j_1+k_2+1} + \dots + r_{j_2+1}} \\
 & + (-1)^3 \sum_{j_2=2}^{n-2} (-1)^{\frac{j_2}{2}} \sum_{r_1=1} \dots \sum_{r_{j_2+1}=1} \frac{(n-1)!}{(r_1 - 1)! \dots (r_{j_2+1} - 1)!} a_{r_1, r_1} \dots \\
 & \quad (r_1 + \dots + r_{j_2+1} = n - 1) \\
 & a_{r_{j_2+1}, r_{j_2+1}} \sum_{j_1=1}^{j_2-1} \frac{r_{j_1+1} + \dots + r_{j_2+1} + 1}{r_{j_1+1} + \dots + r_{j_2+1}} 2 \frac{1}{n - r_1} \prod_{k_1=2}^{j_1} \frac{1}{r_{k_1} + \dots + r_{j_1}} \prod_{k_2=1}^{j_2-j_1} \\
 & \frac{1}{r_{j_1+k_2+1} + \dots + r_{j_2+1}} \Re(B\kappa^2) / \Re(\kappa A) \\
 & + (-1)^2 \sum_{j_2=2}^{n-2} (-1)^{\frac{j_2}{2}} \sum_{r_1=1} \sum_{r_2=1} \dots \sum_{r_{j_2+1}=1} \frac{(n-1)!}{(r_1 - 1)! \dots (r_{j_2+1} - 1)!} a_{r_1, r_1} \dots \\
 & \quad (r_1 + \dots + r_{j_2+1} = n - 1) \\
 & a_{r_{j_2+1}, r_{j_2+1}} \sum_{j_1=1}^{j_2-1} \frac{r_{j_1+1} + \dots + r_{j_2+1} + 1}{r_{j_1+1} + \dots + r_{j_2+1}} 2^{j_2-j_1+1} \frac{1}{n - r_1} \prod_{k_1=2}^{j_1} \frac{1}{r_{k_1} + \dots + r_{j_1}} \\
 & \prod_{k_2=1}^{j_2-j_1} \frac{1}{r_{j_1+k_2+1} + \dots + r_{j_2+1}} \Re(B\kappa^2) / \Re(\kappa A) \\
 & + (-1)^2 \sum_{j_2=2}^{n-2} (-1)^{\frac{j_2+1}{2}} \sum_{r_1=1} \sum_{r_2=1} \dots \sum_{r_{j_2+1}=1} \frac{(n-1)!}{(r_1 - 1)! \dots (r_{j_2+1} - 1)!} a_{r_1, r_1} \dots \\
 & \quad (r_1 + r_2 + \dots + r_{j_2+1} = n - 1) \\
 & a_{r_{j_2+1}, r_{j_2+1}} \sum_{j_1=1}^{j_2-1} \frac{r_{j_1+1} + \dots + r_{j_2+1} + 1}{r_{j_1+1} + \dots + r_{j_2+1}} 2^{j_2-j_1+1} \frac{1}{n - r_1} \prod_{k_1=2}^{j_1} \frac{1}{r_{k_1} + \dots + r_{j_1}}
 \end{aligned}$$

$$\prod_{k_2=1}^{j_2-j_1} \frac{1}{r_{j_1+k_2+1} + \dots + r_{j_2+1}} \Im(B\kappa^2) / \Re(\kappa A). \quad (51)$$

Dieses Verfahren setzen wir fort, so gewinnen wir aus (48), (49), (50) und (51)

das erste Glied von der rechten Seite der Formel (47)

$$\begin{aligned} &= - \sum_{j_1=1}^{n-2} (-1)^{\frac{j_1}{2}} \sum_{r_1=1}^{j_1} \dots \sum_{r_{j_1-1}=1}^{j_1-1} \frac{(n-1)!}{(r_1-1)! \dots (r_{j_1-1}-1)!} a_{r_1, r_1} \dots a_{r_{j_1}, r_{j_1}} \sum_{p=1}^{j_1+1} \alpha_p \beta_p a_{2,1} \\ &\quad (r_1 + \dots + r_{j_1} = n-2) \\ &+ (-1)^2 \sum_{j_2=2}^{n-1} (-1)^{\frac{j_2}{2}} \sum_{r_1=1}^{j_2} \dots \sum_{r_{j_2+1}=1}^{j_2-1} \frac{(n-1)!}{(r_1-1)! \dots (r_{j_2+1}-1)!} \sum_{j_1=1}^{j_2-1} \sum_{h_2=1}^{j_2-j_1+1} a_{r_1, r_1} \dots \\ &\quad (r_1 + \dots + r_{j_2+1} = n) \\ &\quad a_{r_{j_1}, r_{j_1}} a_{r_{j_1+1}, r_{j_1+1}} \dots a_{r_{j_1+h_2}, r_{j_1+h_2-1}} \dots a_{r_{j_2+1}, r_{j_2+1}} \sum_{p=1}^{j_1+1} \alpha_p \beta_p \\ &\quad \prod_{k_2=1}^{j_2-j_1} \frac{1}{r_{j_1+k_2+1} + \dots + r_{j_2+1}} \\ &+ (-1)^3 \sum_{j_2=2}^{n-2} (-1)^{\frac{j_2}{2}} \sum_{r_1=1}^{j_2} \dots \sum_{r_{j_2+1}=1}^{j_2-1} \frac{(n-1)!}{(r_1-1)! \dots (r_{j_2+1}-1)!} a_{r_1, r_1} \dots a_{r_{j_2+1}, r_{j_2+1}} \\ &\quad (r_1 + \dots + r_{j_2+1} = n-1) \\ &\quad \sum_{j_1=1}^{j_2-1} \frac{r_{j_1+1} + \dots + r_{j_2+1} + 1}{r_{j_1+1} + \dots + r_{j_2+1}} 2 \sum_{p=1}^{j_1+1} \alpha_p \beta_p \prod_{k_2=1}^{j_2-j_1} \frac{1}{r_{j_1+k_2+1} + \dots + r_{j_2+1}} \Re(B\kappa^2) / \Re(\kappa A) \\ &+ (-1)^2 \sum_{j_2=2}^{n-2} (-1)^{\frac{j_2}{2}} \sum_{r_1=1}^{j_2} \dots \sum_{r_{j_2+1}=1}^{j_2-1} \frac{(n-1)}{(r_1-1)! \dots (r_{j_2+1}-1)!} a_{r_1, r_1} \dots a_{r_{j_2+1}, r_{j_2+1}} \\ &\quad (r_1 + \dots + r_{j_2+1} = n-1) \\ &\quad \sum_{j_1=1}^{j_2-1} \frac{r_{j_1+1} + \dots + r_{j_2+1} + 1}{r_{j_1+1} + \dots + r_{j_2+1}} 2^{j_2-j_1+1} \sum_{p=1}^{j_1+1} \alpha_p \beta_p \prod_{k_2=1}^{j_2-j_1} \frac{1}{r_{j_1+k_2+1} + \dots + r_{j_2+1}} \Re(B\kappa^2) / \Re(\kappa A) \\ &+ (-1)^2 \sum_{j_2=2}^{n-2} (-1)^{\frac{j_2+1}{2}} \sum_{r_1=1}^{j_2+1} \dots \sum_{r_{j_2+1}=1}^{j_2-1} \frac{(n-1)!}{(r_1-1)! \dots (r_{j_2+1}-1)!} a_{r_1, r_1} \dots a_{r_{j_2+1}, r_{j_2+1}} \\ &\quad (r_1 + \dots + r_{j_2+1} = n-1) \\ &\quad \sum_{j_1=1}^{j_2-1} \frac{r_{j_1+1} + \dots + r_{j_2+1} + 1}{r_{j_1+1} + \dots + r_{j_2+1}} 2^{j_2-j_1+1} \sum_{p=1}^{j_1+1} \alpha_p \beta_p \\ &\quad \prod_{k_2=1}^{j_2-j_1} \frac{1}{r_{j_1+k_2+1} + \dots + r_{j_2+1}} \Im(B\kappa^2) / \Re(\kappa A), \quad (52) \end{aligned}$$

wo

$$\alpha_p = \begin{cases} \prod_{k_1=p}^{j_1} \frac{1}{r_{k_1} + \dots + r_{j_1}} & (\text{wenn } p < j_1 + 1 \text{ ist}) \\ 1 & (\text{wenn } p = j_1 + 1 \text{ ist}) \end{cases} \quad \text{und}$$

$$\beta_p = \begin{cases} 1 & (\text{wenn } p=1 \text{ ist}) \\ \prod_{k_1=1}^{p-1} \frac{1}{n-r_1-\dots-r_{k_1}} & (\text{wenn } p>1 \text{ ist}) \end{cases} \quad \text{ist.}$$

Wir wollen nun das zweite Glied von der rechten Seite der Formel (52) behandeln, und wir nehmen erstens an, dass $h_2=1$ ist.

$$\begin{aligned} & \sum_{j_2=2}^{n-1} (-1)^{\frac{j_2}{2}} \sum_{r_1=1}^{j_2} \dots \sum_{r_{j_2+1}=1}^{j_2} \frac{(n-1)!}{(r_1-1)! \dots (r_{j_2+1}-1)!} \sum_{j_1=1}^{j_2-1} a_{r_1, r_1} \dots \\ & \quad (r_1 + \dots + r_{j_2+1} = n) \\ & a_{r_{j_1}, r_{j_1}} a_{r_{j_1+1}, r_{j_1+1}} \dots a_{r_{j_2+1}, r_{j_2+1}} \sum_{p=1}^{j_1+1} \alpha_p \beta_p \prod_{k_2=1}^{j_2-j_1} \frac{1}{r_{j_1+k_2+1} + \dots + r_{j_2+1}} \\ & = \sum_{j_2=2}^{n-2} (-1)^{\frac{j_2}{2}} \sum_{r_1=1}^{j_2} \dots \sum_{r_{j_1-1}=1}^{j_2} \sum_{r_{j_1+2}=1}^{j_2} \dots \sum_{r_{j_2+1}=1}^{j_2} \frac{(n-1)!}{(r_1-1)! \dots (r_{j_1-1}-1)! (r_{j_1+2}-1)! \dots (r_{j_2+1}-1)!} \\ & \quad (r_1 + \dots + r_{j_1} + r_{j_1+2} + \dots + r_{j_2+1} = n-2) \\ & \sum_{j_1=1}^{j_2-1} a_{r_1, r_1} \dots a_{r_{j_1}, r_{j_1}} a_{r_{j_1+2}, r_{j_1+2}} \dots a_{r_{j_2+1}, r_{j_2+1}} \sum_{p=1}^{j_1+1} \alpha_p \beta_p \prod_{k_2=1}^{j_2-j_1} \frac{1}{r_{j_1+k_2+1} + \dots + r_{j_2+1}} a_{2,1} \\ & + \sum_{j_2=2}^{n-2} (-1)^{\frac{j_2}{2}} \sum_{r_1=1}^{j_2} \dots \sum_{r_{j_1-1}=1}^{j_2} \sum_{r_{j_1+1}=3}^{j_2} \dots \sum_{r_{j_2+1}=1}^{j_2} \frac{(n-1)!}{(r_1-1)! \dots (r_{j_1-1}-1)! (r_{j_1+1}-1)! \dots (r_{j_2+1}-1)!} \\ & \quad (r_1 + \dots + r_{j_2+1} = n, r_{j_1+1} \geq 3) \\ & \sum_{j_1=1}^{j_2-1} a_{r_1, r_1} \dots a_{r_{j_1}, r_{j_1}} a_{r_{j_1+1}, r_{j_1+1}} \dots a_{r_{j_2+1}, r_{j_2+1}} \sum_{p=1}^{j_1+1} \alpha_p \beta_p \prod_{k_2=1}^{j_2-j_1} \frac{1}{r_{j_1+k_2+1} + \dots + r_{j_2+1}}. \end{aligned}$$

Es ist aber

das erste Glied von der rechten Seite der obigen Formel

$$\begin{aligned} & = \sum_{j_2=2}^{n-2} (-1)^{\frac{j_2}{2}} \sum_{r_1=1}^{j_2} \dots \sum_{r_{j_2}=1}^{j_2} \frac{(n-1)!}{(r_1-1)! \dots (r_{j_2}-1)!} \sum_{j_1=1}^{j_2-1} a_{r_1, r_1} \\ & \quad (r_1 + \dots + r_{j_2} = n-2) \\ & a_{r_{j_1}, r_{j_1}} a_{r_{j_1+1}, r_{j_1+1}} \dots a_{r_{j_2}, r_{j_2}} \sum_{p=1}^{j_1+1} \alpha_p \beta_p \prod_{k_2=1}^{j_2-j_1} \frac{1}{r_{j_1+k_2+1} + \dots + r_{j_2}} a_{2,1}. \end{aligned} \quad (53)$$

Das zweite Glied von der rechten Seite der obigen Formel

$$\begin{aligned} & = \sum_{j_2=2}^{n-2} (-1)^{\frac{j_2}{2}} \sum_{r_1=1}^{j_2} \dots \sum_{r_{j_1+1}=3}^{j_2} \dots \sum_{r_{j_2+1}=1}^{j_2} \frac{(n-1)!}{(r_1-1)! \dots (r_{j_1-1}-1)! (r_{j_1+1}-1)! \dots (r_{j_2+1}-1)!} \\ & \quad (r_1 + \dots + r_{j_2+1} = n, r_{j_1+1} \geq 3) \\ & \sum_{j_1=1}^{j_2-1} a_{r_1, r_1} \dots a_{r_{j_1}, r_{j_1}} a_{r_{j_1+2}, r_{j_1+2}} \dots a_{r_{j_2+1}, r_{j_2+1}} \times - \left[\sum_{j_3=1}^{r_{j_1+1}-1} (-1)^{\frac{j_3}{2}} \sum_{s_1=1}^{j_3} \dots \sum_{s_{j_3+1}=1}^{j_3} \right. \\ & \quad \left. (s_1 + \dots + s_{j_3+1} = r_{j_1+1}) \right] \end{aligned}$$

$$\begin{aligned}
& \frac{(r_{j_1+1}-1)!}{(s_1-1)! \cdots (s_{j_3+1}-1)!} \sum_{h_3=1}^{j_3+1} a_{s_1, r_1} \cdots a_{s_{h_3}, r_{h_3}-1} \cdots a_{s_{j_3+1}, r_{j_3+1}} \prod_{k_3=1}^{j_3} \frac{1}{s_{k_3+1} + \cdots + s_{j_3+1}} \\
& + 2 {}_{r_{j_1+1}} C_1 \Re(B\kappa^2) a_{r_{j_1+1}-1, r_{j_1+1}-1} / \Re(\kappa A) + {}_{r_{j_1+1}} C_1 \left\{ \sum_{j_3=1}^{r_{j_1+1}-2} (-1)^{\frac{j_3}{2}} \Re(B\kappa^2) \right. \\
& + \left. \sum_{j_3=1}^{r_{j_1+1}-2} (-1)^{\frac{j_3+1}{2}} \Im(B\kappa^2) \right\} \sum_{s_1=1} \cdots \sum_{s_{j_3+1}=1} \frac{(r_{j_1+1}-2)! 2^{j_3+1}}{(s_1-1)! \cdots (s_{j_3+1}-1)!} a_{s_1, r_1} \cdots a_{s_{j_3+1}, r_{j_3+1}} \\
& \quad (s_1 + \cdots + s_{j_3+1} = r_{j_1+1}-1) \\
& \prod_{k_3=1}^{j_3} \frac{1}{s_{k_3+1} + \cdots + s_{j_3+1}} / \Re(\kappa A) \left] \sum_{p=1}^{j_1+1} \alpha_p \beta_p \prod_{k_2=1}^{j_2-j_1} \frac{1}{r_{j_1+k_2+1} + \cdots + r_{j_2+1}} \right. \\
& = - \sum_{j_3=3}^{n-1} (-1)^{\frac{j_3}{2}} \sum_{r_1=1} \cdots \sum_{r_{j_3+1}=1} \frac{(n-1)!}{(r_1-1)! \cdots (r_{j_3+1}-1)!} \sum_{j_2=2}^{j_3-1} \sum_{j_1=1}^{j_2-1} a_{r_1, r_1} \cdots \\
& \quad (r_1 + \cdots + r_{j_3+1} = n) \\
& a_{r_{j_2}, r_{j_2}} \sum_{h_3=1}^{j_3-j_2+1} a_{r_{j_2+1}, r_{j_2+1}} \cdots a_{r_{j_2+h_3}, r_{j_2+h_3}-1} \cdots a_{r_{j_3+1}, r_{j_3+1}} \sum_{p=1}^{j_1+1} \alpha_p \beta_p \prod_{k_2=1}^{j_2-j_1} \frac{1}{r_{j_1+k_2+1} + \cdots + r_{j_2}} \\
& \prod_{k_3=1}^{j_3-j_2} \frac{1}{r_{j_2+k_3+1} + \cdots + r_{j_3+1}} \\
& + \sum_{j_3=3}^{n-2} (-1)^{\frac{j_3}{2}} \sum_{r_1=1} \cdots \sum_{r_{j_3+1}=1} \frac{(n-1)!}{(r_1-1)! \cdots (r_{j_3+1}-1)!} a_{r_1, r_1} \cdots a_{r_{j_3+1}, r_{j_3+1}} \\
& \quad (r_1 + \cdots + r_{j_3+1} = n-1) \\
& \sum_{j_2=2}^{j_3-1} \sum_{j_1=1}^{j_2-1} \frac{r_{j_2+1} + \cdots + r_{j_3+1} + 1}{r_{j_2+1} + \cdots + r_{j_3+1}} 2 \sum_{p=1}^{j_1+1} \alpha_p \beta_p \prod_{k_2=1}^{j_2-j_1} \frac{1}{r_{j_1+k_2+1} + \cdots + r_{j_2}} \\
& \prod_{k_3=1}^{j_3-j_2} \frac{1}{r_{j_2+k_3+1} + \cdots + r_{j_3+1}} \Re(B\kappa^2) / \Re(\kappa A) \\
& - \sum_{j_3=3}^{n-2} (-1)^{\frac{j_3}{2}} \sum_{r_1=1} \cdots \sum_{r_{j_3+1}=1} \frac{(n-1)!}{(r_1-1)! \cdots (r_{j_3+1}-1)!} a_{r_1, r_1} \cdots a_{r_{j_3+1}, r_{j_3+1}} \\
& \quad (r_1 + \cdots + r_{j_3+1} = n-1) \\
& \sum_{j_2=2}^{j_3-1} \sum_{j_1=1}^{j_2-1} \frac{r_{j_2+1} + \cdots + r_{j_3+1} + 1}{r_{j_2+1} + \cdots + r_{j_3+1}} 2^{j_3-j_2+1} \sum_{p=1}^{j_1+1} \alpha_p \beta_p \prod_{k_2=1}^{j_2-j_1} \frac{1}{r_{j_1+k_2+1} + \cdots + r_{j_2}} \\
& \prod_{k_3=1}^{j_3-j_2} \frac{1}{r_{j_2+k_3+1} + \cdots + r_{j_3+1}} \Re(B\kappa^2) / \Re(\kappa A) \\
& - \sum_{j_3=3}^{n-2} (-1)^{\frac{j_3+1}{2}} \sum_{r_1=1} \cdots \sum_{r_{j_3+1}=1} \frac{(n-1)!}{(r_1-1)! \cdots (r_{j_3+1}-1)!} a_{r_1, r_1} \cdots a_{r_{j_3+1}, r_{j_3+1}} \\
& \quad (r_1 + \cdots + r_{j_3+1} = n-1)
\end{aligned}$$

$$\frac{\sum_{j_2=2}^{j_3-1} \sum_{j_1=1}^{j_2-1} \frac{r_{j_2+1} + \dots + r_{j_3+1} + 1}{r_{j_2+1} + \dots + r_{j_3+1}} 2^{j_3-j_2+1} \sum_{p=1}^{j_1+1} \alpha_p \beta_p \prod_{k_2=1}^{j_2-j_1} \frac{1}{r_{j_1+k_2} + \dots + r_{j_2}}}{\prod_{k_3=1}^{j_3-j_2} \frac{1}{r_{j_2+k_3+1} + \dots + r_{j_3+1}}} \mathfrak{N}(B\kappa^2) / \mathfrak{N}(\kappa A). \quad (54)$$

Indem wir ganze analoge Berechnungen für $h_2 = 2, \dots, j_2 - j_1 + 1$ ausführen, gewinnen wir aus (52), (53) und (54)

das zweite Glied von der rechten Seite der Formel (52)

$$\begin{aligned} &= (-1)^2 \sum_{j_2=2}^{n-1} (-1)^{\frac{j_2}{2}} \sum_{r_1=1}^{j_2} \dots \sum_{r_{j_2}=1}^{(n-1)!} \frac{(n-1)!}{(r_1-1)! \dots (r_{j_2}-1)!} a_{r_1, r_1} \dots \\ &\quad (r_1 + \dots + r_{j_2} = n-2) \\ & a_{r_{j_2}, r_{j_2}} \sum_{j_1=1}^{j_2-1} \sum_{p_1=1}^{j_1+1} \alpha_{p_1}^1 \beta_{p_1}^1 \sum_{p_2=1}^{j_2+1} \alpha_{p_2}^2 \beta_{p_2}^2 a_{2,1} + (-1)^3 \sum_{j_3=3}^{n-1} (-1)^{\frac{j_3}{2}} \sum_{r_1=1}^{j_3} \dots \sum_{r_{j_3+1}=1}^{j_3} \\ &\quad (r_1 + \dots + r_{j_3+1} = n) \\ & \frac{(n-1)!}{(r_1-1)! \dots (r_{j_3+1}-1)!} \sum_{j_2=2}^{j_3-1} \sum_{j_1=1}^{j_2-1} a_{r_1, r_1} \dots a_{r_{j_2}, r_{j_2}} \sum_{h_3=1}^{j_3-j_2+1} a_{r_{j_2+1}, r_{j_2+1}} \dots \\ & a_{r_{j_2+h_3}, r_{j_2+h_3}} \dots a_{r_{j_3+1}, r_{j_3+1}} \sum_{p_1=1}^{j_1+1} \alpha_{p_1}^1 \beta_{p_1}^1 \sum_{p_2=1}^{j_2+1} \alpha_{p_2}^2 \beta_{p_2}^2 \prod_{k_3=1}^{j_3-j_2} \frac{1}{r_{j_2+k_3+1} + \dots + r_{j_3+1}} \\ & + (-1)^4 \sum_{j_3=3}^{n-2} (-1)^{\frac{j_3}{2}} \sum_{r_1=1}^{j_3} \dots \sum_{r_{j_3+1}=1}^{(n-1)!} \frac{(n-1)!}{(r_1-1)! \dots (r_{j_3+1}-1)!} a_{r_1, r_1} \dots \\ &\quad (r_1 + \dots + r_{j_3+1} = n-1) \\ & a_{r_{j_3+1}, r_{j_3+1}} \sum_{j_2=2}^{j_3-1} \sum_{j_1=1}^{j_2-1} \frac{r_{j_2+1} + \dots + r_{j_3+1} + 1}{r_{j_2+1} + \dots + r_{j_3+1}} 2 \sum_{p_1=1}^{j_1+1} \alpha_{p_1}^1 \beta_{p_1}^1 \sum_{p_2=1}^{j_2+1} \alpha_{p_2}^2 \beta_{p_2}^2 \\ & \prod_{k_3=1}^{j_3-j_2} \frac{1}{r_{j_2+k_3+1} + \dots + r_{j_3+1}} \mathfrak{N}(B\kappa^2) / \mathfrak{N}(\kappa A) \\ & + (-1)^3 \sum_{j_3=3}^{n-2} (-1)^{\frac{j_3}{2}} \sum_{r_1=1}^{j_3} \dots \sum_{r_{j_3+1}=1}^{(n-1)!} \frac{(n-1)!}{(r_1-1)! \dots (r_{j_3+1}-1)!} a_{r_1, r_1} \dots \\ &\quad (r_1 + \dots + r_{j_3+1} = n-1) \\ & a_{r_{j_3+1}, r_{j_3+1}} \sum_{j_2=2}^{j_3-1} \sum_{j_1=1}^{j_2-1} \frac{r_{j_2+1} + \dots + r_{j_3+1} + 1}{r_{j_2+1} + \dots + r_{j_3+1}} 2^{j_3-j_2+1} \sum_{p_1=1}^{j_1+1} \alpha_{p_1}^1 \beta_{p_1}^1 \sum_{p_2=1}^{j_2+1} \alpha_{p_2}^2 \beta_{p_2}^2 \\ & \prod_{k_3=1}^{j_3-j_2} \frac{1}{r_{j_2+k_3+1} + \dots + r_{j_3+1}} \mathfrak{N}(B\kappa^2) / \mathfrak{N}(\kappa A) \\ & + (-1)^3 \sum_{j_3=3}^{n-2} (-1)^{\frac{j_3}{2}} \sum_{r_1=1}^{j_3+1} \dots \sum_{r_{j_3+1}=1}^{(n-1)!} \frac{(n-1)!}{(r_1-1)! \dots (r_{j_3+1}-1)!} a_{r_1, r_1} \dots \\ &\quad (r_1 + \dots + r_{j_3+1} = n-1) \end{aligned}$$

$$a_{r_{j_3+1}, r_{j_3+1}} \frac{\sum'_{j_2=2}^{j_3-1} \sum'_{j_1=1}^{j_2-1} \frac{r_{j_2+1} + \dots + r_{j_3+1} + 1}{r_{j_2+1} + \dots + r_{j_3+1}}}{\prod_{k_3=1}^{j_3-j_2} \frac{1}{r_{j_2+k_3+1} + \dots + r_{j_3+1}}} \Im(B\kappa^2) / \Re(\kappa A), \quad (55)$$

wo

$$\begin{aligned} \alpha_{p_1}^1 &= \begin{cases} \prod_{k_1=p_1}^{j_1} \frac{1}{r_{k_1} + \dots + r_{j_1}} & (\text{wenn } 1 \leq p_1 \leq j_1 \text{ ist}) \\ 1 & (\text{wenn } p_1 = j_1 + 1 \text{ ist}) \end{cases} \quad \text{und} \\ \beta_{p_1}^1 &= \begin{cases} 1 & (\text{wenn } p_1 = 1 \text{ ist}) \\ \prod_{k_1=1}^{p_1-1} \frac{1}{n - r_1 - \dots - r_{k_1}} & (\text{wenn } 2 \leq p_1 \leq j_1 + 1 \text{ ist}) \end{cases} \quad \text{und} \\ \alpha_{p_2}^2 &= \begin{cases} \prod_{k_2=p_2}^{j_2-j_1} \frac{1}{r_{j_1+k_2} + \dots + r_{j_2}} & (\text{wenn } 1 \leq p_2 \leq j_2 - j_1 \text{ ist}) \\ 1 & (\text{wenn } p_2 = j_2 - j_1 + 1 \text{ ist}) \end{cases} \quad \text{und} \\ \beta_{p_2}^2 &= \begin{cases} 1 & (\text{wenn } p_2 = 1 \text{ ist}) \\ \prod_{k_2=1}^{p_2-1} \frac{1}{n - r_1 - \dots - r_{j_1+k_2}} & (\text{wenn } 2 \leq p_2 \leq j_2 - j_1 + 1 \text{ ist}) \end{cases} \quad \text{ist.} \end{aligned}$$

Dieses Verfahren setzen wir fort, so gewinnen wir endlich

$$\begin{aligned} a_{n, n-1} &= \sum_{l=2}^{n-2} (-1)^l \sum'_{j_l=l}^{n-2} (-1)^{\frac{j_l}{2}} \sum_{r_1=1}^{j_l} \dots \sum_{r_{j_l+1}=1}^{j_l} \frac{(n-1)!}{(r_1-1)! \dots (r_{j_l}-1)!} a_{r_1, r_1} \dots \\ &\quad (r_1 + \dots + r_{j_l} = n-2) \\ a_{r_{j_l+1}, r_{j_l+1}} &\frac{\sum'_{j_{l-1}=1}^{j_l-1} \sum'_{j_{l-2}=1}^{j_{l-1}-1} \dots \sum'_{j_2=1}^{j_{l-1}-1} \sum'_{j_1=1}^{j_2-1} \sum_{p_1=1}^{j_1+1} \alpha_{p_1}^1 \beta_{p_1}^1 \sum_{p_2=1}^{j_2+1} \alpha_{p_2}^2 \beta_{p_2}^2 \dots \sum_{p_l=1}^{j_l+1} \alpha_{p_l}^l \beta_{p_l}^l a_{2,1}}{r_{j_{l-1}+1} + \dots + r_{j_l+1} = n-1} \\ &- \sum'_{j_1=1}^{n-2} (-1)^{\frac{j_1}{2}} \sum_{r_1=1}^{j_1} \dots \sum_{r_{j_1+1}=1}^{j_1} \frac{(n-1)!}{(r_1-1)! \dots (r_{j_1+1}-1)!} a_{r_1, r_1} \dots a_{r_{j_1+1}, r_{j_1+1}} \\ &\quad (r_1 + \dots + r_{j_1+1} = n-1) \\ &\frac{r_1 + r_2 + \dots + r_{j_1+1} + 1}{r_1 + r_2 + \dots + r_{j_1+1}} 2^{j_1+1} \prod_{k_1=1}^{j_1} \frac{1}{r_{k_1+1} + \dots + r_{j_1+1}} \Re(B\kappa^2) / \Re(\kappa A) \\ &+ \sum_{l=2}^{n-2} (-1)^l \sum'_{j_l=l}^{n-2} (-1)^{\frac{j_l}{2}} \sum_{r_1=1}^{j_l} \dots \sum_{r_{j_l+1}=1}^{j_l} \frac{(n-1)!}{(r_1-1)! \dots (r_{j_l+1}-1)!} a_{r_1, r_1} \dots \\ &\quad (r_1 + \dots + r_{j_l+1} = n-1) \\ a_{r_{j_l+1}, r_{j_l+1}} &\frac{\sum'_{j_{l-1}=1}^{j_l-1} \sum'_{j_{l-2}=1}^{j_{l-1}-1} \dots \sum'_{j_2=1}^{j_{l-1}-1} \sum'_{j_1=1}^{j_2-1} \frac{r_{j_{l-1}+1} + \dots + r_{j_l+1} + 1}{r_{j_{l-1}+1} + \dots + r_{j_l+1}}}{r_{j_{l-1}+1} + \dots + r_{j_l+1} = n-1} 2^{j_l-j_{l-1}+1} \end{aligned}$$

$$\begin{aligned}
 & \sum_{p_1=1}^{j_1+1} \alpha_{p_1}^1 \beta_{p_1}^1 \sum_{p_2=1}^{j_2+1} \alpha_{p_2}^2 \beta_{p_2}^2 \cdots \sum_{p_l=1}^{j_l+1} \alpha_{p_l}^l \beta_{p_l}^l \prod_{k_l=1}^{j_l-j_{l-1}} \frac{1}{r_{j_{l-1}+k_l+1} + \cdots + r_{j_l+1}} \Re(B_{\kappa^2}) / \Re(\kappa A) \\
 & - \sum_{j_1=1}^{n-2} (-1)^{\frac{j_1+1}{2}} \sum_{r_1=1} \cdots \sum_{r_{j_1+1}=1} \frac{(n-1)!}{(r_1-1)! \cdots (r_{j_1+1}-1)!} a_{r_1, r_1} \cdots a_{r_{j_1+1}, r_{j_1+1}} \\
 & \quad (r_1 + \cdots + r_{j_1+1} = n-1) \\
 & \frac{r_1 + \cdots + r_{j_1+1} + 1}{r_1 + \cdots + r_{j_1+1}} 2^{j_1+1} \prod_{k_1=1}^{j_1} \frac{1}{r_{k_1+1} + \cdots + r_{j_1+1}} \Im(B_{\kappa^2}) / \Re(\kappa A) \\
 & + \sum_{i=2}^{n-2} (-1)^i \sum_{j_l=i}^{n-2} (-1)^{\frac{j_l+1}{2}} \sum_{r_1=1} \cdots \sum_{r_{j_l+1}=1} \frac{(n-1)!}{(r_1-1)! \cdots (r_{j_l+1}-1)!} a_{r_1, r_1} \cdots \\
 & \quad (r_1 + \cdots + r_{j_l+1} = n-1) \\
 & a_{r_{j_l+1}, r_{j_l+1}} \sum_{j_{l-1}=1}^{j_l-1} \sum_{j_{l-2}=1}^{j_{l-1}-1} \cdots \sum_{j_2=1}^{j_3-1} \sum_{j_1=1}^{j_2-1} \frac{r_{j_{l-1}+1} + \cdots + r_{j_l+1} + 1}{r_{j_{l-1}+1} + \cdots + r_{j_l+1}} 2^{j_l-j_{l-1}+1} \\
 & \sum_{p_1=1}^{j_1+1} \alpha_{p_1}^1 \beta_{p_1}^1 \sum_{p_2=1}^{j_2+1} \alpha_{p_2}^2 \beta_{p_2}^2 \cdots \sum_{p_l=1}^{j_l+1} \alpha_{p_l}^l \beta_{p_l}^l \prod_{k_l=1}^{j_l-j_{l-1}} \frac{1}{r_{j_{l-1}+k_l+1} + \cdots + r_{j_l+1}} \Im(B_{\kappa^2}) / \Re(\kappa A) \\
 & + (-1)^2 \sum_{j_1=1}^{n-2} (-1)^{\frac{j_1}{2}} \sum_{r_1=1} \cdots \sum_{r_{j_1+1}=1} \frac{(n-1)!}{(r_1-1)! \cdots (r_{j_1+1}-1)!} a_{r_1, r_1} \cdots \\
 & \quad (r_1 + \cdots + r_{j_1+1} = n-1) \\
 & a_{r_{j_1+1}, r_{j_1+1}} \frac{r_1 + r_2 + \cdots + r_{j_1+1} + 1}{r_1 + r_2 + \cdots + r_{j_1+1}} 2 \Re(B_{\kappa^2}) / \Re(\kappa A) + \sum_{i=2}^{n-2} (-1)^i \sum_{j_l=i}^{n-2} (-1)^{\frac{j_l}{2}} \\
 & \sum_{r_1=1} \cdots \sum_{r_{j_l+1}=1} \frac{(n-1)!}{(r_1-1)! \cdots (r_{j_l+1}-1)!} a_{r_1, r_1} \cdots a_{r_{j_l+1}, r_{j_l+1}} \\
 & \quad (r_1 + \cdots + r_{j_l+1} = n-1) \\
 & \sum_{j_{l-1}=1}^{j_l-1} \sum_{j_{l-2}=1}^{j_{l-1}-1} \cdots \sum_{j_2=1}^{j_3-1} \sum_{j_1=1}^{j_2-1} \frac{r_{j_{l-1}+1} + \cdots + r_{j_l+1} + 1}{r_{j_{l-1}+1} + \cdots + r_{j_l+1}} 2 \sum_{p_1=1}^{j_1+1} \alpha_{p_1}^1 \beta_{p_1}^1 \sum_{p_2=1}^{j_2+1} \alpha_{p_2}^2 \beta_{p_2}^2 \cdots \\
 & \sum_{p_l=1}^{j_l+1} \alpha_{p_l}^l \beta_{p_l}^l \prod_{k_l=1}^{j_l-j_{l-1}} \frac{1}{r_{j_{l-1}+k_l+1} + \cdots + r_{j_l+1}} \Re(B_{\kappa^2}) / \Re(\kappa A) \\
 & - \sum_{j_1=1}^{n-2} (-1)^{\frac{j_1}{2}} \sum_{r_1=1} \cdots \sum_{r_{j_1+1}=1} \frac{(n-1)!}{(r_1-1)! \cdots (r_{j_1+1}-1)!} a_{r_1, r_1} \cdots \\
 & \quad (r_1 + \cdots + r_{j_1+1} = n-2) \\
 & a_{r_{j_1+1}, r_{j_1+1}} \sum_{p_1=1}^{j_1+1} \alpha_{p_1}^1 \beta_{p_1}^1 a_{2,1}, ,
 \end{aligned} \tag{56}$$

wo

$$\alpha_{p_1}^1 = \begin{cases} \prod_{k_1=p_1}^{j_1} \frac{1}{r_{k_1} + \cdots + r_{j_1}} & (\text{wenn } 1 \leq p_1 \leq j_1 \text{ ist}) \\ 1 & (\text{wenn } p_1 = j_1 + 1 \text{ ist}) \end{cases} \quad \text{und} \quad \left. \vphantom{\alpha_{p_1}^1} \right\}$$

$$\left. \begin{aligned}
 \beta_{p_1}^1 &= \begin{cases} 1 & (\text{wenn } p_1 = 1 \text{ ist}) \\ \prod_{k_1=1}^{p_1-1} \frac{1}{n-r_1-\dots-r_{k_1}} & (\text{wenn } 2 \leq p_1 \leq j_1+1 \text{ ist}) \end{cases} & \text{und} \\
 \alpha_{p_s}^s &= \begin{cases} \prod_{k_s=p_s}^{j_s-j_{s-1}-1} \frac{1}{r_{j_{s-1}+k_s}+\dots+r_{j_s}} & (\text{wenn } 1 \leq p_s \leq j_s-j_{s-1} \text{ ist}) \\ 1 & (\text{wenn } p_s = j_s-j_{s-1}+1 \text{ ist}) \end{cases} & \text{und} \\
 \beta_{p_s}^s &= \begin{cases} 1 & (\text{wenn } p_s = 1) \\ \prod_{k_s=1}^{p_s-1} \frac{1}{n-r_1-\dots-r_{j_{s-1}+k_s}} & (\text{wenn } 2 \leq p_s \leq j_s-j_{s-1}+1 \text{ ist}) \end{cases} & \text{ist.}
 \end{aligned} \right\} \quad (57)$$

(für $s=2, 3, \dots, n-2$)

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